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STUDY OF THE EFFECTS OF ATMOSPHERIC TURBULENCE ON LASER COMMUNICATIONS SYSTEMS

by

William E. Webb, Project Director

and

Kermit H. George

Interim Report on Contract NAS8-25562

March 1971

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STUDY OF ATMOSPHERIC EFFECTS ON LASER COMMUNICATIONS SYSTEMS

Volume II

ATMOSPHERIC EFFECTS ON WAVE PROPAGATION AT 10.6 MICRONS

by

William E. Webb, Project Director

and

Kermit H. George

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CHAPTER I

INTRODUCTION

The atmosphere is an inhomogeneous non-isotropic media which is usually in a state of turbulence. The atmosphere is characterized by its temperature, wind velocity, and humidity. The variations of these parameters comprise non-stationary random processes and as a result the fluctuation of the index of refraction of the media is also a non-stationary random process. Electromagnetic radiation at optical wave lengths propagating through the atmosphere will be greatly affected by the random fluctuation in the index of refraction. This causes a very complicated scattering to occur which results in amplitude and phase variations of the wave. Clearly, these fluctuations are also of a random nature.

The distortion induced by the atmosphere on the propagating wave is of great concern in the development of optical tracking and communication systems since performance can be seriously degraded due to this effect. For example, amplitude fluctuations cause the signal-to-noise ratio to be reduced in incoherent detection systems. Atmosphere distortion tends to reduce the coherence of the wave which in turn reduces the effective power level of the signal. This will also decrease the signal-to-noise ratio. Loss of coherence is a problem in systems utilizing coherent detection where heterodyne action must be achieved. Wave front tilt due to phase variations of the wave induces errors in optical tracking receivers since they view this tilt as an apparent angle of arrival.

An important aspect in the design of optical systems that is de-

pendent on a knowledge of atmospheric distortion is the size of the optical components. Atmospheric effects can be somewhat overcome by employing large receiving optics which will tend to average out the wave fluctuations. This advantage is limited by the high expense and difficulty in fabrication of such components. The designer must then choose optimum size components which require that he have a thorough knowledge of the atmospheric problem. It is clear that there is a pressing need for an accurate mathematical model of the atmosphere.

Statistical methods must be employed to analyze this problem since the wave fluctuations are random processes. The first step in developing a statistical model is to determine the probability density function of the random process. Theoretical considerations have predicted a log-normal distribution for the amplitude and normal for the phase. This theory has been experimentally verified for visible wave lengths, but results of current investigations in the infrared region of 10.6 microns have been inconsistent.

D. L. Fried[1] has made scintillation measurements at this wave length over a 1 km. range using a point detector. His results do not confirm the hypothesis that intensity scintillation is log-normally distributed. He suggests that this may be a genuine feature of 10.6 micron scintillation but draws no definite conclusion since detector noise and nonlinearity problems in taking measurements could have influenced his results.

Richard Kerr[2] of the Oregon Graduate Research Center has conducted multiwave length laser propagation studies over a mile path and claims confirmation of log-normal statistics for wave lengths of 4880Å and 10.6 microns. In addition Fitzmaurice, Bufton, and Minott[3] have also concluded that scintillation at 10.6 micron fits the log-normal model. Their

work was done over a 2.4 km. path. Both investigators used point source detection.

The effect of the atmosphere on 10.6 micron propagation is important since the popular CO_2 laser emits radiation at this wave length. This type laser is attractive for application in optical systems due to its high efficiency and high output power capability. In addition, the atmospheric effect at this wave length is much less than that at visible wave lengths.

The purpose of this study is experimentally to investigate the statisitical properties of scintillation and the signal-to-noise ratio of heterodyne detection for a CO_2 laser beam propagated over a 3.2 km. path. Both scintillation and heterodyne measurements have been made for a variety of receiving aperture sizes ranging from two to ten cm.

A brief discussion of the theory which is referred to in current literature is presented in Chapter II. The necessary statistical concepts are introduced before a qualitative description of atmospheric turbulence is given. Finally, the physical significance of aperture averaging is discussed.

Chapter III gives a detailed description of the experiment. Described is the equipment, its alignment and check out as well as a discussion on the techniques used to make the measurements.

The handling and reducing of the data is given in Chapter IV. This includes a discussion on the conversion of analog data to digital form for direct use on a digital computer. An outline of the computer program which reduces the data is presented. The theory used to calculate aperture effects is also given.

Chapter V is concerned with interpreting the reduced data to de-

termine if the hypothesis of the log-normal distribution for intensity scintillation is valid for this wave length. This chapter also includes results of calculations for the refractive index structure constant with and without aperture averaging corrections.

Chapter VI contains the summary and conclusions of this study as well as recommendations for further study. A complete documentation for the computer program is given in the Appendix.

CHAPTER II

THEORY

A. Statistical Concepts

It is necessary to give a discussion on pertinent statistical concepts as a prelude to presenting a qualitative discussion on the theoretical aspects of the atmospheric problem.

The random processes are described in terms of parameters which are random variables. The value of any such function at a fixed instant of time is a random variable having definite probability density function. The process may further be described by its auto-covariance function at times t_1 and t_2

$$AC\{f(t_1), f(t_2)\} = \langle [f(t_1) - \langle f(t_1) \rangle][f(t_2) - \langle f(t_2) \rangle] \rangle \qquad 2\text{-}1$$

where $\langle\ \rangle$ indicates an ensemble average. The auto-covariance function reduces to the correlation function

$$B[f(t_1), f(t_2)] = \langle f(t_1) f(t_2) \rangle \qquad 2\text{-}2$$

for processes where the mean value is zero. The auto-covariance function characterizes the mutual relation between the fluctuations at different instants of time. The mean value of the random variable can be a constant or can change with time. Similarly, the auto-covariance function can either depend only on the difference between the times t_1 and t_2 or else it can depend on the positions of the points on the time axis. The first case would occur when the statistical relation between the

fluctuations of the variable at different instants of time does not change with time. A random function is called stationary if its mean value does not depend on time and its auto-covariance function depends only on the difference between observation times.

The mean value of the meteorological parameters of the atmosphere such as temperature, wind velocity, and humidity undergo comparatively slow and smooth changes. These variables are non-stationary processes if the definition of stationarity is strictly applied. It is difficult to determine which changes in the fluctuation are to be regarded as slow changes in the mean and which are to be regarded as slow fluctuations of the function.

To avoid this difficulty and to describe random functions which have the above characteristics, the structure function is used instead of the correlation function. This function was first introduced by Kolmogorov[4,5]. The basic idea behind this method is to use the difference function

$$F_\tau(t) = f(t+\tau) - f(t) \qquad 2\text{-}3$$

instead of the non-stationary function f(t). For values of τ which are not too large, slow changes in the function f(t) do not affect the value of the difference function which means that it can be considered a stationary random function. The function f(t) is called a random function with stationary increments. To derive an expression for the structure function consider the transformation of the correlation function for $F_\tau(t_1)$ and $F_\tau(t_2)$:

$$B(t_1, t_2) = \langle [F_\tau(t_1) F_\tau(t_2)] \rangle \qquad 2\text{-}4$$

$$B(t_1, t_2) = \left\langle [f(t_1+\tau) - f(t_1)][f(t_2+\tau) - f(t_2)] \right\rangle \qquad 2\text{-}5$$

Using the algebraic identity

$$(a-b)(c-d) = \frac{1}{2}\left[(a-d)^2 + (b-c)^2 - (a-c)^2 - (b-d)^2\right] \qquad 2\text{-}6$$

we have

$$B(t_1, t_2) = \frac{1}{2}\left\langle [f(t_1+\tau) - f(t_2)]^2 \right\rangle + \frac{1}{2}\left\langle [f(t_1) - f(t_2+\tau)]^2 \right\rangle$$
$$- \frac{1}{2}\left\langle [f(t_1+\tau) - f(t_2+\tau)]^2 \right\rangle - \frac{1}{2}\left\langle [f(t_1) - f(t_2)]^2 \right\rangle \qquad 2\text{-}7$$

Thus the correlation function is expressed as a linear combination of functions of the form

$$D_f(t_i, t_j) = \left\langle [f(t_i) - f(t_j)]^2 \right\rangle \qquad 2\text{-}8$$

which is called the structure function of the random process. The form of the structure function more commonly used

$$D_f(\tau) = \left\langle [f(t+\tau) - f(t)]^2 \right\rangle \qquad 2\text{-}9$$

is the basic characteristic of a random process with stationary increments. The value of $D(\tau)$ characterizes the intensity of those fluctuations of $f(t)$ which are smaller than or are comparable with τ.

B. Nature of Atmospheric Turbulence

The statistical theory of turbulence was initiated in the works of Friedmann and Keller[6]. This theory was greatly amended in 1941 when A. N. Kolmogorov and A. M. Obukhov[6] established the laws which characterize the basic properties of the microstructure of turbulent

flow at very large Reynolds numbers. The following discussion is drawn from Tatarski's[6] work on the Kolmogorov theory.

Consider the atmosphere to be a viscous fluid in a state of laminar flow. This flow can be characterized by its viscosity ν, velocity v, and the characteristic length L. The quantity L characterizes the dimensions of the flow as a whole and arises from the boundary conditions of the fluid dynamics problem. This laminar flow will be stable if the Reynolds number

$$R = \frac{VL}{\nu} \qquad 2\text{-}10$$

does not exceed a certain critical value.

Suppose that for some reason a velocity fluctuation occurs in a region of size ℓ of the initial laminar flow. The value of Reynolds number will increase and the laminar motion will lose stability. The result of this instability is the formation of a secondary flow or eddies within L which will have their own Reynolds number R_ℓ. As the Reynolds number for the overall flow is increased, R_ℓ will increase causing the secondary flow to break up into smaller scale eddies. These new eddies now give energy to even smaller eddies and the process continues until an eddy with a Reynolds number less than the critical value is formed. The atmospheric turbulence can be considered as consisting of many circulating eddies having different flow characteristics. Eddies are usually described in terms of an inner and outer scale of turbulence. These are measures of the characteristic sizes of the smallest and largest eddies which exist at the time of interest. Figure 1 may aid in visualizing this process. The outer scale is the physical dimension of the largest eddy. The inner scale of turbulence is roughly the size of the smallest

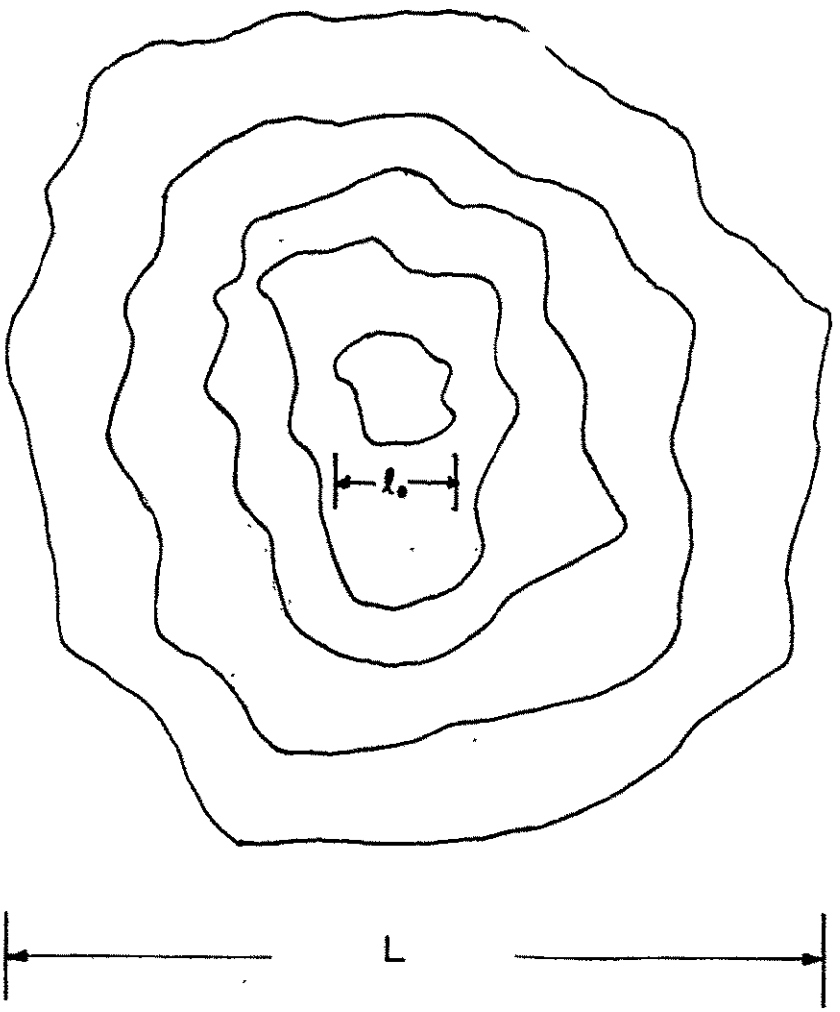


Figure 1. Visualization of Microstructure of Turbulence.

stable eddy. It can be more precisely defined in terms of a characteristic of the longitudinal velocity structure function

$$D_{rr} = \left\langle [V(\bar{r}_1) - V(\bar{r}_2)]^2 \right\rangle \qquad 2\text{-}11$$

where $V(\bar{r}_1)$ is the projection of the velocity at the point $\bar{r}_1$ along the direction of $\bar{r}$, and $V(\bar{r}_2)$ is the same quantity at the point

$$\bar{r}_2 = \bar{r}_1 + \bar{r}$$

For $r \ll \ell_o$ $\qquad D_{rr} = ar^2 \qquad 2\text{-}12$

and for $r \gg \ell_o$ $\qquad D_{rr} = c^2 r^{2/3} \qquad 2\text{-}13$

For r on the order of L the structure function saturates. The inner scale of turbulence ℓ_o is then defined mathematically as the value for D_{rr} where the functions in equations 2-12 and 2-13 intersect.

Each eddy or cell in the field of turbulence can be considered locally isotropic and homogeneous, and as a result it will have a certain index of refraction, which we will assume to be constant throughout the cell. The index of refraction will in general differ from cell to cell. As an electromagnetic wave passes through each cell two things occur: First, the phase of the wave is advanced or retarded in a random manner due to the index of refraction of the cell. Secondly, the wave is scattered due to the interfaces between the cells. This causes the intensity of the beam to be distributed at random after traveling through a large number of cells.

From the model of the atmosphere just developed some insight as to the statistical characteristics of phase and amplitude variation can be

gained. Since the random variations in phase add to each other, the Central Limit Theorem[7] can be applied to predict that the distribution of the phase across the wave front is normal. The amplitude of the wave has a distinctly different character. After each refraction the intensity is the product of the intensity before scattering and the variation due to refraction. The intensity variations are then due to a product of probabilities. If the Central Limit Theorem is applied to the logarithm of the intensities, they will be normally distributed. This leads to a log-normal probability density function for the amplitude distribution. Because of this, it is customary to describe waves propagating through the atmosphere in terms of their phase $\phi(r,t)$ and log-amplitude $L_a(r,t)$, where $L_a(r,t)$ is given by

$$L_a(r,t) = \ln \left\{\frac{A(r,t)}{\overline{A}(r,t)}\right\} \qquad 2\text{-}14$$

or in terms on intensities

$$L_a(r,t) = \frac{1}{2} \ln \left\{\frac{I(r,t)}{\overline{I}(r,t)}\right\} \qquad 2\text{-}15$$

where $\overline{A}$ and $\overline{I}$ are mean values. Using these equations the complex representations of the wave becomes

$$\overline{A}(r,t) \exp [L(r,t) + j\phi(r,t)] \qquad 2\text{-}16$$

C. Aperture Averaging

Collection of light with large aperture optical systems tends to average out atmospherically induced intensity and phase fluctuations. This causes a smaller variance for both and alters the statistical properties of the intensity variation.

Before discussing the principles of aperture averaging it is necessary to define correlation distance r_o. Consider the intensity at two points separated by a distance of r. For r equal zero the correlation function will be unity. As r increases, the correlation function decreases with zero as its lower bound. How rapidly the correlation function decreases with increasing r, is related to the strength of atmospheric turbulence. r_o is defined as the distance at which the intensity variations of the two points are no longer correlated. r_o could also be defined as the distance at which the variations become statistically independent. r_o usually varies from a few millimeters for very strong turbulence to several centimeters for mild turbulence.

Consider the aperture shown in Figure 2 to be the aperture of an optical system which collects and focuses light from a diverging beam. Let the light intensity across the illuminated aperture be divided into n finite circles of radius r_o. The intensity within these circles will be assumed to be highly correlated. The collection and focusing process can be thought of as adding the intensity contribution of each circle on the aperture in the focal plane. Averaging of the intensity fluctuations of each circle across the aperture will occur at the focus if the diameter of the aperture is much larger than r_o such that it contains many circles.

The intensity fluctuation in the circles of radius r_o are log-normally distributed. Since the aperture adds the intensities, it will also add the probability density functions. The Central Limit Theorem can then be applied to predict that the intensity variations at the focus should be normally distributed. In order to apply the Central Limit Theorem, we must assume that the average intensities of the circles are

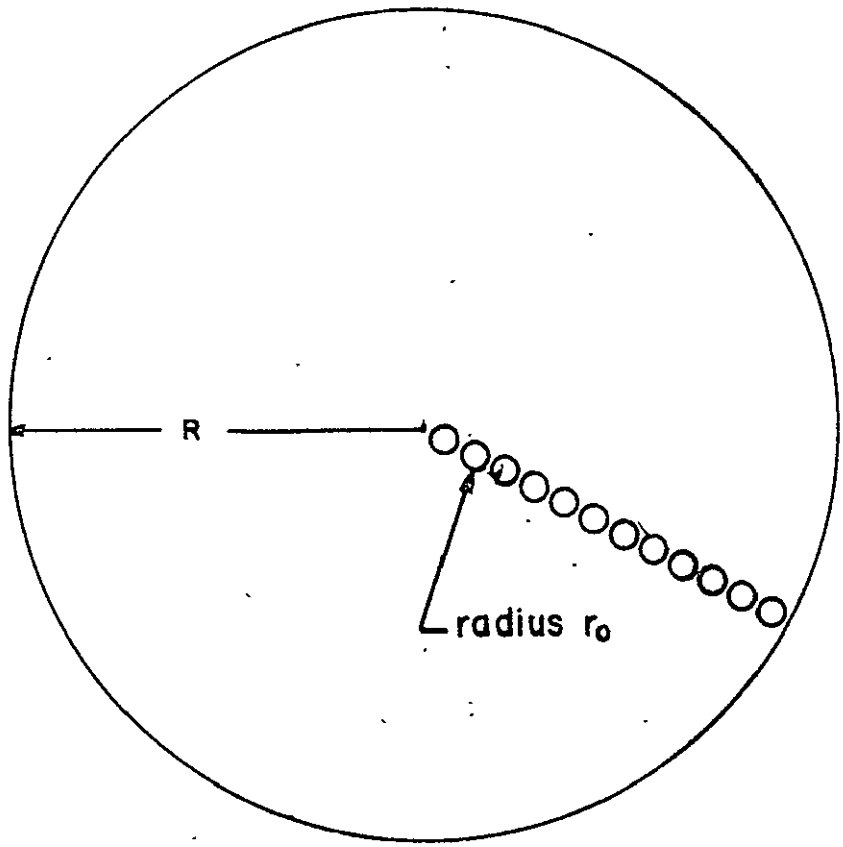


Figure 2. Illustration of the Concept of Aperture Averaging.

of the same order of magnitude across the aperture and that the variance from circle to circle should not change significantly.

CHAPTER III

EXPERIMENTAL

A. Description of Equipment

The experimental measurements were made during the Summer of 1969 at the Marshall Space Flight Center's optical range located on Redstone Arsenal near Huntsville, Alabama.

The transmitting system was located in an astronomical observatory on the crest of Madkin Mountain. The receiving and recording systems were located in the Astrionics Laboratory complex in a special building equipped with a large mirror periscope so that the optical equipment could be conveniently placed on the ground level yet have a clear optical path to the mountain. The height of the periscope was about 15 feet above ground level. The optical path extended in a southwesterly direction for a distance of 3.2 km. The transmitter was about 220 meters above the receiver so that the optical path was at an angle of 4° with the horizontal. Except for a few small buildings and a parking lot paved with bituminous material, the optical path lay mostly over wooded terrain.

The transmitter and receiver were constructed by Minneapolis Honeywell Corporation for Marshall Space Flight Center and have been described in the literature[8]. The transmitter consisted of a 5 watt CO_2 flowing gas laser with a 10 cm. off axis cassegrainian collimator as shown in Figure 3. The laser was designed to have good short and long term frequency stability. This was accomplished in part by constructing the cavity of the low expansion material cervit, which has an expansion

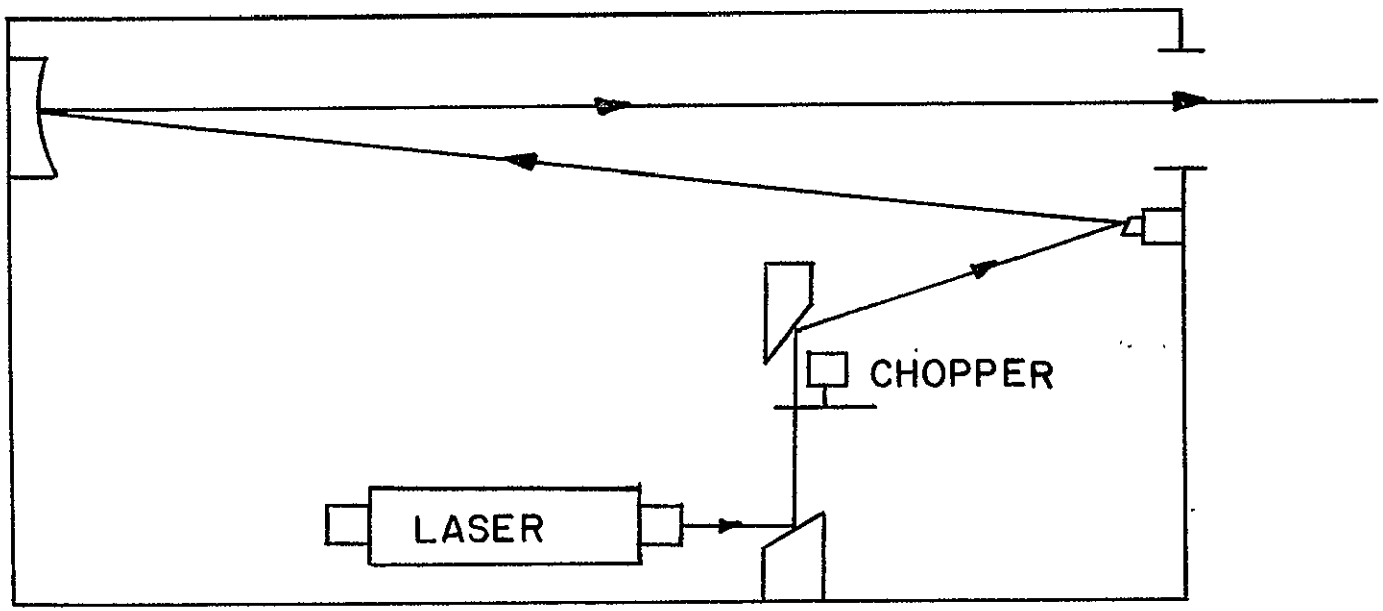


Figure 3. Side View of Transmitter.

coefficient of less than 1×10^{-7} (1/°C). To further stabilize the laser, water held at a constant temperature to better than 0.1°C was continuously circulated through the cervit yoke. The laser in the transmitter was frequency modulated by applying the modulation voltage to a piezoelectric cylinder on which one of the end mirrors was mounted. The other end mirror consisted of an Irtran output coupler that was also attached to a piezoelectric cylinder, which provided laser transition selection. A DC bias was applied to the cylinder to select the desired transition. In addition, the transmitter included a mechanical chopper that was originally intended to be used for alignment purposes. The transmitter unit also contained the necessary electronics to produce a modulation voltage for both carrier or direct modulation. A block diagram of the transmitting unit is given in Figure 4.

The receiving unit was housed in a cabinet identical to that of the transmitter as shown in Figure 5. The receiver consisted of a 10 cm. off axis cassegrainian telescope, a local oscillator laser identical to that of the transmitter, combining optics, and a mercury doped cadmium telluride detector. This is an alloy detector having a spectral response between 8 and 14 microns. The detector was cryogenic and required an operating temperature of 77°K, which was obtained by using liquid nitrogen. The operation of the receiving unit can be described with reference to Figure 6. The transmitter and the local oscillator signal are made spatially colinear by means of a germanium beam splitter and combined on the surface of the mercury doped cadmium telluride detector. The local oscillator frequency is offset by 10 Mhz from the transmitting laser. The 10 Mhz beat frequency produced by the detector is amplified with a 10 Mhz center frequency, intermediate frequency amplifier. This

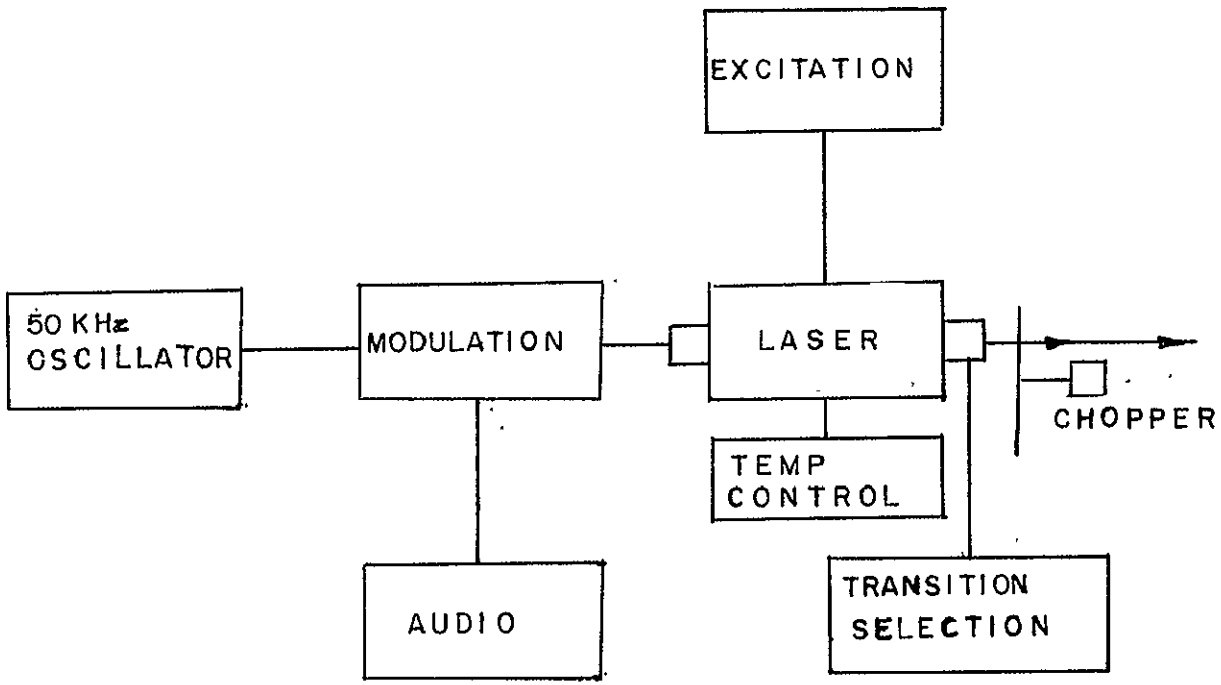


Figure 4. Block Diagram of Transmitter.

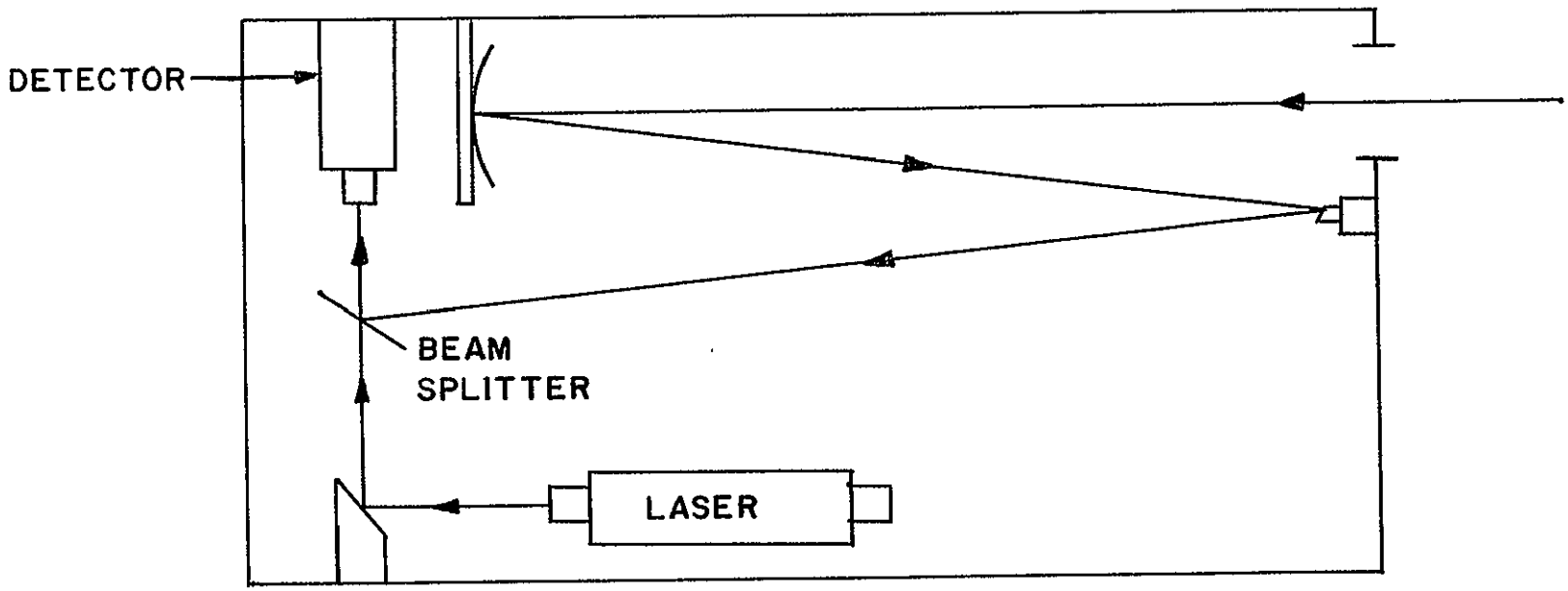


Figure 5. Side View of Receiver.

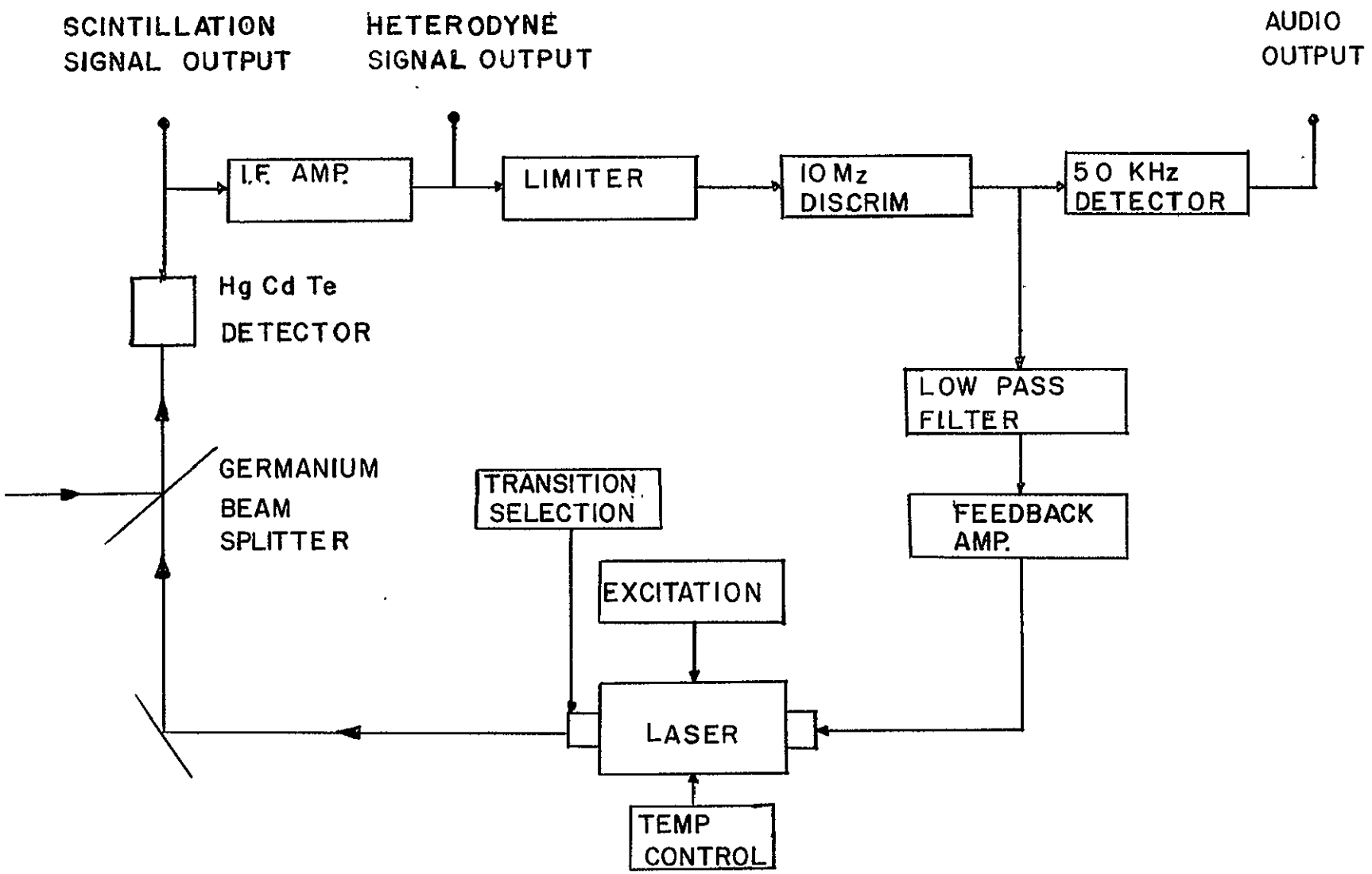


Figure 6. Block Diagram of Receiver.

amplifier has a 2 Mhz bandwidth and a 110 db gain. The intermediate frequency amplifier is followed by a limiter that eliminates amplitude variations. A 10 Mhz discriminator provides an error signal for the local laser feedback loop which consists of a low pass filter and a feedback amplifier. The feedback amplifier drives a piezoelectric cylinder on which one end mirror of the laser is mounted. This provides automatic frequency control of the local oscillator laser.

The data acquisition system was located at the receiver terminal and consisted of an Ampex 14 channel analog F.M. tape recorder, a variable gain AC amplifier with good low frequency response, and two oscilloscopes used for monitoring purposes. Figure 7 gives a block diagram of this system. A spectrum analyzer was also available to check the 10 Mhz beat signal in the receiver.

The monitoring of both the input to the amplifier and the recorder was necessary to insure that they were operating within their linear range. Especially critical was the input level to the recorder, since its linear range for input voltage was ±1 volt. To be safe we operated within ±.5 volt.

B. System Alignment

It was necessary to measure the laser and amplifier noise of the system to insure that it would not have a significant effect on measurements made through the atmosphere. Noise measurements were made with the transmitter and receiver placed a few feet apart with the local laser in the receiver turned off, so that only noise contributions from the transmitter laser, detector and receiver electronics would be present. The system was then operated in the same manner over a 100 meter path through an enclosed tunnel one meter in diameter. The noise

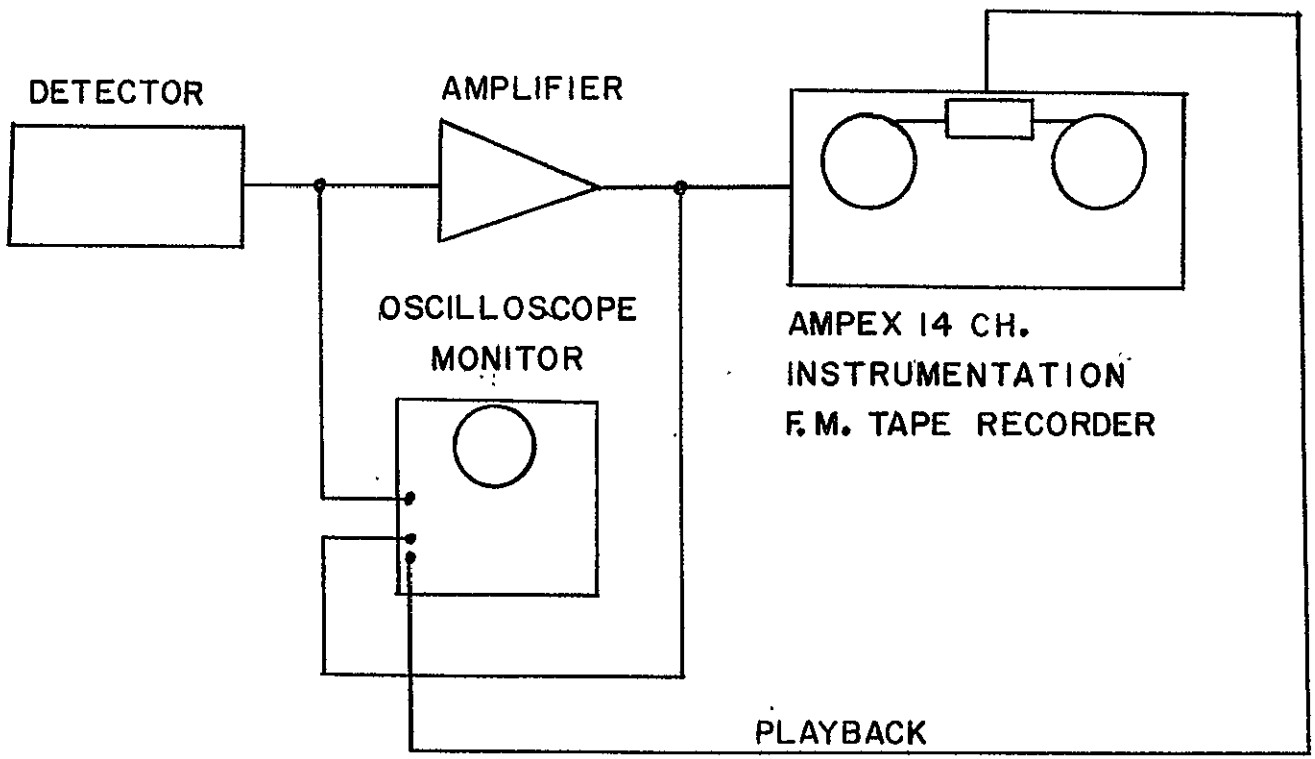


Figure 7. Block Diagram of Data Acquisition System.

of the local oscillator laser was determined by operating it into the detector in the absence of an incoming signal. The noise in all cases was found to be sufficiently low so that it would be negligible compared to the expected variation due to atmospheric scintillation. The linearity of the mercury cadmium telluride detector was determined by noting changes in DC voltage output for different power levels. The laser power was measured on one side of the germanium beam splitter using a Coherent Radiation Laboratories power meter and the DC variation in the detector was measured by a digital voltmeter. For incident power levels less than 300 mw., the detector output was found to be linear.

Alignment of the system over the 3.2 km. path proved to be a difficult task. Our first attempt was to bore sight a 60 power telescope mounted on the transmitter case, with the output beam. The idea was to aim the transmitting unit on the mountain at the laboratory periscope. This method worked over the 100 meter tunnel quite well, but the bore sight became misaligned when the transmitter was transported to the mountain. The second method employed to align the system involved the use of two visible lasers. The transmitter unit, now mounted on the observatory telescope stand on Madkin Mountain, was aligned by placing an argon laser directly in place of the receiving unit in the laboratory. The bright beam could easily be detected by the eye at the transmitting terminal. The position of the argon laser was adjusted until its beam was intercepted by the objective of the transmitter unit. The optical system of the transmitter was then adjusted so that the visible laser was focused onto the output aperture at the transmitter laser. Using heat sensitive paper as a position indicator for the infrared beam, the visible light and the invisible beam were made to coincide at two points in the optical system. Alignment of the receiving unit was accomplished

in a similar manner except that a visible laser could not be mounted in the same position as the now aligned transmitter unit. A small helium neon laser was mounted with a telescope on a tripod and located as near to the transmitter as possible. The telescope was then bore sighted to the helium neon laser. The laser-telescope arrangement was pointed by locating the top periscope mirror at the laboratory. A corner reflector located at this mirror enabled a more precise aim as the reflection of the red light could be seen with the telescope. The visible beam was then focused onto the detector in the receiver. With minor adjustments to the transmitter laser mount, close alignment was attained for the system.

C. Measurement Procedure

Scintillation measurements were made with the receiver laser inoperative so that only light from the transmitter and from the sun's reflection off the observatory dome were intercepted by the receiver. This background light was cause for great concern since accurate measurements of the variations of the laser light could not be made in its presence. Since we could not filter out this unwanted light, it was necessary to record it. To accomplish this scintillation measurements were made by chopping the transmitter beam at 90 Hz by means of the mechanical chopper located in the transmitter. Figure 8 shows a sample of this chopped signal.

Scintillation measurements were made in 90 second segments. A written log was kept on each data run giving the date and time of day, analog tape number, the channel number, aperture size, and weather conditions such as temperature, wind velocity and humidity. The intensity signal for each data run was read on the analog tape into marked time slots. One

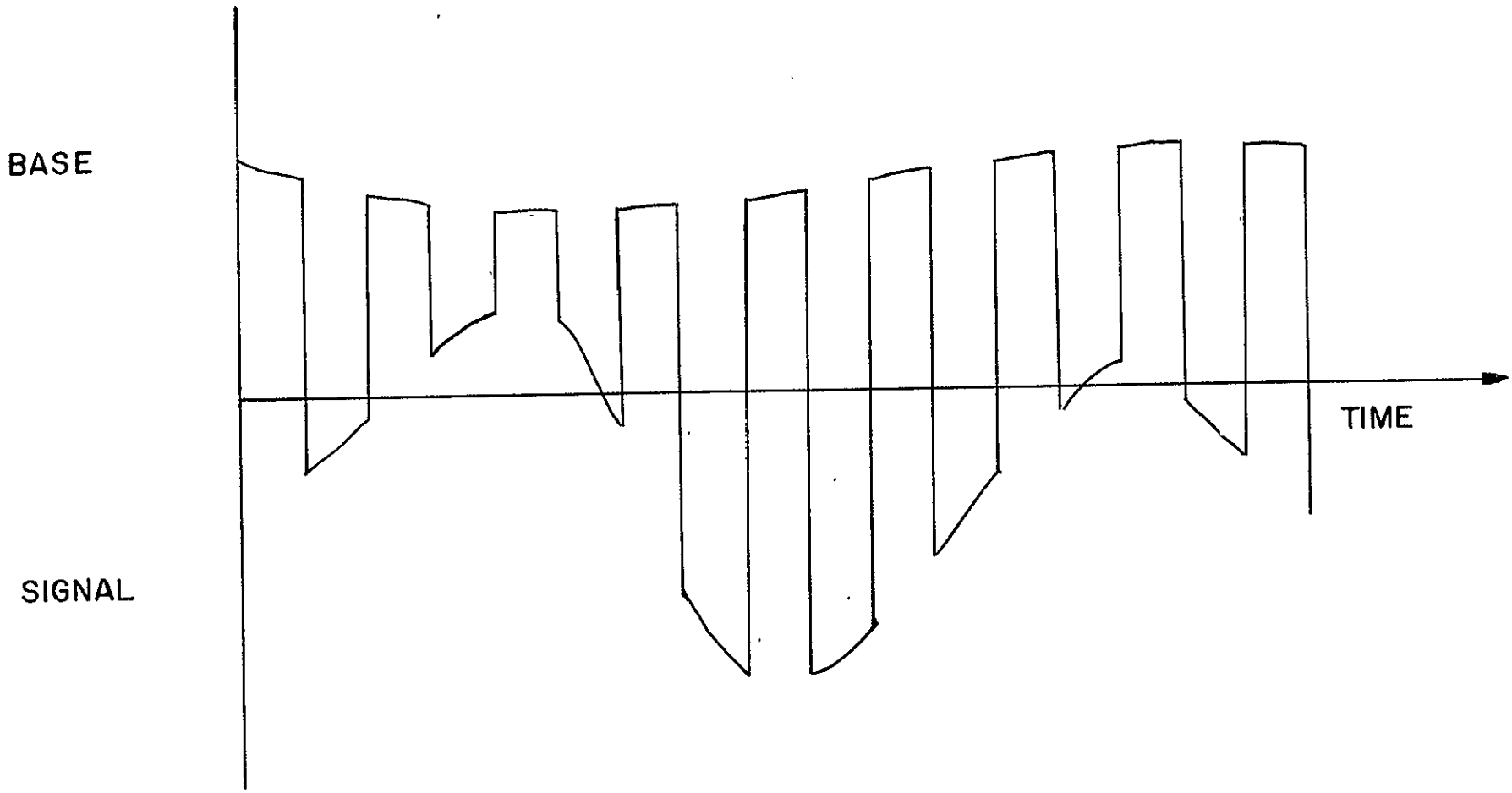


Figure 8. Waveform of Signal Recorded for Scintillation Measurements.

of the channels was marked every 90 seconds with an audio tone code so that approximately 33 slots were available on each data channel. When sufficient signal was being received, data runs were made in succession using five different size apertures. The aperture size varied from two to ten cm. in diameter. The runs were made in succession to insure that atmospheric conditions remained constant over each set of five runs.

Since the signal level for small aperture sizes was below that for the larger apertures, it was necessary to adjust the AC amplifier from one run to another using the monitor oscilloscope. Also, it was important to keep the dewar on the detector filled with liquid nitrogen so that the detector temperature would remain constant.

Measurements of the signal to noise ratio and distortion were made on the communication system operating over the optical range. The signal-to-noise ratio of the heterodyne action was measured at the receiver by extracting the 10 Mhz beat note between the received signal and the local oscillator after it had passed through one stage of amplification. The signal was detected with a simple diode circuit and the resulting voltage recorded by the data acquisition system. These measurements were also made for different aperture sizes.

CHAPTER IV

DATA REDUCTION

A. Analog-to-Digital Conversion

All measurements were recorded on analog tape. In order to process this data it was necessary to convert it to a digital form suitable for computer input. The very large quantity of data collected necessitated electronic conversion to digital magnetic tape which could then be read directly into the computer.

The first step in the conversion from analog-to-digital form consisted of recording a timing signal on a reserved channel of the analog tape. Eight standard time signals are broadcast by Marshall Space Flight Center's Computation Laboratory on a frequency of 226.5 Mhz. The time signals are subcarrier multiplexed with pilot frequencies between 2.3 KHz and 70.0 KHz. The signal chosen was a rectangular pulse with a one-second repetition rate broadcast on a subcarrier frequency of 52.5 KHz. This signal is designated as 100/1000 AMR-D-5, and is coded with the time of day in Greenwich Mean Time by pulse width modulation. This timing signal had no relation to the actual time the data was recorded.

The analog tapes were then read into a cathode-ray-oscillograph. Four data channels, the timing channel and the marker channel were recorded simultaneously. The oscillograms were inspected and the sections of the tape to be digitized were selected. At this time bad data was identified and eliminated. The starting and stopping time for each time interval selected was read from the timing channel and recorded on the Computation Laboratories instruction forms. The analog-to-digital converter system was set to convert only those time intervals selected by

reading the timing channel. Approximately 30 one-minute segments were selected from each channel. As the running time for the analog tape was about 45 minutes, about two-thirds of the total data recorded on a tape channel was converted.

The analog-to-digital conversion was performed by Marshall Space Flight Center's Computation Laboratory using an Astrodata type converter. Five data channels were digitized simultaneously. The five channels were sampled alternately at a rate of 5000 samples per second which amounted to sampling each channel at 1000 samples per second. The resulting binary coding was recorded on seven channel digital tape in a multiplexed format.

The digital tape format consisted of a ten bit binary word so that 2048 levels were available to represent analog signal levels between -.5 volts and +.5 volts. Since seven track tape was used, each sample required two tape characters consisting of a ten bit word plus the sign bit, blank bit and two parity bits. The data was recorded in records of 2004 tape characters. The first four tape characters contained the time as read from the timing channel at which the first sample was taken. The remaining 2000 characters contained 1000 sample points, 200 from each channel, alternating between channels. Subsequent records were written until the segment was completed, at which time an end of file mark was written; therefore, each file on the digital tape contained one time slice or data run from the analog tape. In addition, the first record of each file was an identification record of 24 tape characters which contained the tape number and other information.

B. Description of Computer Program

A program was written for an IBM-360-50 computer to reduce the data

stored on magnetic tapes. This program reads data from the tape, changes the binary format to a fortran compatible form, then calls its various subroutines to perform the analysis. The principle problems which were encountered in writing this program concerned formatting the data for the computer and extracting the actual light intensity signal from the modulated square wave. When the signal was extracted it was either stored for spectral analysis or is used to construct a histogram. A Fourier transform subroutine or statistical subroutine was then called to analyze the signal.

Development of a routine to extract the desired signal proved to be somewhat difficult since the sampling rate during digitization could not be accurately synchronized with the period of the square wave. The sampling rate of 1 KHz and the chopping rate of 90 Hz should yield approximately 10 samples per cycle of the square wave. In actuality, the number of samples per cycle varied between ten and twelve due to the sampling rate not being an integral multiple of the square wave frequency. The routine was designed to determine whether a particular data sample was a base point (i.e., from the part of the square wave when the laser beam was blocked by the chopper) or a signal point (when power was being received from the laser beam). The problem was further complicated by the fact that the rise and fall times of the square wave were non-negligible so that about one percent of the data points were sampled during the switching transient and should be neglected. In addition, some of the data contained an occasional noise spike which should be eliminated. It was decided that the elimination of these spikes would not adversely effect the validity of the analysis so provisions for eliminating them were also included in the program.

The extraction routine operates basically as follows: A pre-processing routine reads the data from the magnetic tape and stores a record containing 200 data points into a common array. To begin the analysis twenty data points from the array are selected and their maximum and minimum value computed. Two limits, L_1 and L_2 are then set by the relations

$$L_1 = A_{max} - P_1(A_{max} - A_{min}) \qquad 4\text{-}1$$

$$L_2 = A_{min} - P_2(A_{max} - A_{min}) \qquad 4\text{-}2$$

where A_{max} and A_{min} are the maximum and minimum values of the first twenty points, and P_1 and P_2 are constants between zero and one half. Since the signal was inverted when it was recorded on the analog tape, the base line is greater than the signal, hence a particular point greater than L_1 is considered a base point, if it is less than L_2 it is considered a signal point. Points lying between L_1 and L_2 are assumed to be from the transient portion of the wave form and are neglected. The routine takes each point successively and determines if it is a base point, a signal point or neither. As a preliminary to processing, the first twenty points are scanned and the beginning of a base line segment of the wave form is found. Then new limits are set on the next 15 points and they are scanned and grouped into three arrays, a base line segment, a signal segment and a second base line segment. Each array may contain up to ten points. At this time another routine is used to compute the signal amplitude of the square wave for the group of signal points (as will be described later). The second group of base points is transferred into the first array, new limits are set using the next ten data points,

and a new group of signal points and base points are found to fill the second and third arrays, and finally their amplitudes computed. This process is continued until the 200 points from the first record have been used. At this time the routine pauses while the next record is read in. Processing then continues until the number of records called for (up to 300) have been processed.

This routine also contains several checks to handle possible irregularities in the data. During the search for either base or signal points if more than ten consecutive points are found, the routine will request that the next record be read in, which means the remainder of the bad record is discarded. Also if the number of unsuccessful scans while in the base or signal search phase exceeds ten, the routine will enter an error recycle phase. In this phase the routine skips 20 points and resumes processing as if it were at the beginning of a new run. If the error recycle phase is entered five times in a given record, a request for the next record will be executed. When a new record is requested due to an irregularity in the record being processed the routine again treats it as it does the first record of a new run. In addition to the above, if three or less base or signal points are found in a given search, the error recycle phase is entered.

During processing, a record is kept of each irregularity encountered and this information is printed in tabular form when the processing of a run is completed. If an excessive number of irregularities occurs in a given run, the results of that run must be suspected.

The three arrays containing base and signal points found by the extraction routine are passed into a routine that determines the actual amplitude of the signal. Three methods for computing the amplitude were

tried: The first method took to base line for a group of signal points as the average of all the points in the group of base points preceding it and the ones following it. This is equivalent to considering the background light during the time when the laser beam intensity was being recorded to be the average of the background light recorded during the half-cycle immediately preceding the signal and the half-cycle immediately following it. The difference between the signal point and the average base line was taken as the amplitude of the laser beam at that instance.

The second method of amplitude calculation considered was to reconstruct the base line by fitting a least-mean-square curve to the base points. This method produced erratic results and also required additional computer time and was abandoned.

In the third approach, the difference in the first signal point in a group and the last base point preceding it is taken as the signal amplitude. The difference in the last signal point in the group and the first base point following it gives a second amplitude. This method yields only two amplitudes per cycle but has the advantage that they are evenly spaced. Since the difference in time between signal and base points is very small, this method eliminates the problem of the unknown base line over the signal interval.

The computer program contained both the first and third methods of signal amplitudes calculations. The method to be used was selected by a parameter read during execution. The values computed by this routine were either stored in an array to be used in the spectral analysis, or used to construct a histogram. Histograms are sometimes referred to as being the probability density function for discrete variables.

The final segment of the program consists of the analysis routines.

The statistical routines accept the histogram for the intensity fluctuation which has been generated and computes the scintillation statistics. The routine computes and lists the corresponding value of the log-amplitude as defined by

$$L_i = \frac{1}{2} \ln [I_i/\bar{I}] \qquad 4\text{-}3$$

where L and I_i are the log-amplitude and the intensity for the ith class interval and $\bar{I}$ is the mean intensity. The frequency for each class and the cumulative probability are also listed. The routine also calculates the mean, standard deviation, skewness, and kurtosis for both the intensity and log-amplitude distribution. A chi-square test that checks the intensity distribution for a normal fit and the log-amplitude for a log-normal fit is also included. Appendix A includes a typical computer print out of the statistical analysis.

The program includes routines to perform a spectral analysis on the intensity scintillation. In calculating the scintillation spectrum 2^8 (8192) values of the beam intensity were extracted from the raw data using the preprocessing routines described previously. These values represent the intensity fluctuations of the beam sampled at 180 Hz rate. The spectrum of the sampled data was computed using the Cooley-Tukey algorithm[9] for fast Fourier transforms. The resulting spectra were displayed by means of a plot routine. In cases where data points are discarded due to irregularities in the extraction routine, the omitted points were replaced with zero values. The purpose of this was to preserve the phase relationship of the signal, which is important in the integration operation of the Fourier transform.

C. Program Check and Parameter Adjustment

To test the program several records were read from a magnetic tape and printed out for inspection. Each sample was classified either as a signal point, or a base point, or as a point from the transient portion of the waveform. This classification was purely subjective, yet in inspecting the data there was usually no questions as to how a point should be categorized. The same data was then read into the computer. A print out of all the pertinent variables at each decision branch of the extraction routine was obtained. The program was then run several times varying the parameters P_1 and P_2 in equations 4-1 and 4-2 between runs and the results compared with the subjective analysis. Data having irregularities was also processed and the results compared to determine whether or not a segment should be omitted. Using these comparisons as a criteria, the parameters P_1 and P_2 were set at 0.05 and 0.10 respectively. Therefore a point within 5% of the maximum base point or 10% of the minimum signal point would be retained while points between these limits were discarded.

The statistical routines were checked out with sets of numbers which had a known probability distribution function. A routine readily available in the IBM Scientific Subroutine Package for the IBM 360 computer was used to generate a large set of normally distributed numbers. These numbers were then processed by the statistics routines in the same manner as the intensity fluctions were to be processed. The part of the routine which tests for the normal distribution fit gave very positive results, and that for the log-normal fit gave negative results. The test routine was then altered to generate numbers with a log-normal distribution. The results of the statistical analysis of this data strongly indicated a

log-normal fit. A description of the log-normal generator is given in Appendix B.

D. Atmospheric Structure Constant and Aperture Averaging

An important parameter in the statistical model for atmospheric studies used in current literature is the refractive index structure constant C_n^2, defined as a constant of proportionality in the relation

$$D_n = \left\langle [n_1(r_1) - n_2(r_2)]^2 \right\rangle = C_n^2 \, r^{2/3}$$

$$r = r_2 - r_1 \qquad 4\text{-}4$$

where D_n is called the structure function and is a measure of the deviation of the index of refraction at two points separated by a distance r. C_n is actually a measure of the strength of the turbulence Fried[10] gives a relation involving C_n for an infinite spherical wave propagating a distance z in a turbulent atmosphere. The relation is

$$C_\ell(0) = .124 \, K^{7/6} \, Z^{11/6} \, C_n^2 \qquad 4\text{-}5$$

where $C_\ell(0)$ is the log-amplitude variance. Equation 4-5 can be used directly to obtain C_n^2 by noting that the standard deviation of the log-amplitude distribution which we have computed is the square root of $C_\ell(0)$.

A finite receiving aperture has the effect of averaging the intensity fluctuating from various parts of the wave front thereby reducing the variance of the scintillation.

The effect of aperture averaging can be allowed for by using relations developed by Fried[11,12] viz.

$$\sigma_s^2 = [\frac{\pi}{4} D^2]^2 \theta C_I(0) \qquad 4\text{-}6$$

Where σ_s is the signal variance which corresponds to the square of the standard deviation of the intensity fluctuation, D is the diameter of the receiving aperture, and θ is an aperture averaging factor given by:

$$\theta = \frac{16}{\pi D^2} \int_0^D \rho d\rho \frac{\exp[4C_\ell(\rho)]-1}{\exp[4C_\ell(0)]-1} H(\rho/D) \qquad 4\text{-}7$$

$H(\rho/D)$ is the optical transfer function of a circular aperture

$$H(\rho/D) = \cos^{-1}(\rho/D) - \rho/D\ [1-(\rho/D)^2]^{1/2} \qquad 4\text{-}8$$

and $C_\ell(\rho)$ is log-amplitude covariance given by:

$$C_\ell(\rho) = C_\ell(0) \sum_{n=0}^{\infty} [a_n + b_n \left(\frac{k\rho^2}{4z}\right)][\left(\frac{k\rho^2}{4z}\right)/(2n)!]$$

$$- 7.53034 \{\frac{k\rho^2}{4z}\}^{5/6} \qquad 4\text{-}9$$

In the last expression a_n and b_n are the expansion coefficients for the modified confluent hypergeometric function and are given by the recursion relations

$$a_n = -a_{n-1} \{(2n - 23/6)(2n - 17/6)/(2n-1)(2n)\}$$

$$b_n = -b_{n-1} \{(2n - 11/6)(2n - 17/6)(2n-1)/(2n)(2n+1)^2\} \qquad 4\text{-}10$$

where

$$a_o = 1 \qquad b_o = 6.84209$$

The intensity variance $C_I(0)$ can be related to the log-variance by:

$$C_I(0) = I_o^2 \, [\exp(4C_\ell(0)) - 1] \tag{4-11}$$

Equation 4-11 specifies $C_\ell(0)$ in terms of σ_s^2 and I_o^2. I_o and σ_s are the mean and standard deviation of the recorded intensity. Combining the above equations we have:

$$\left(\frac{\sigma_s}{I_o}\right)^2 = \pi D^2 \, [\exp[4C_\ell(0)] - 1] \times \int_o^D \rho d\rho \, \frac{\exp[4f(\frac{k\rho}{4z}) \, C_\ell(0)]-1}{\exp[4C_\ell(0)]-1} \, H(\rho D) \tag{4-12}$$

where $f(k\rho/4z)$ is the summation given in equation 4-9. Since σ_s/I_o is an experimentally determined constant, equation 4-12 is an integral equation for $C_\ell(0)$. A special computer program was written to solve equation 4-12. The technique used is to evaluate the integral in equation 4-12 for a number of trial values of $C_\ell(0)$ using a fourth order Runga-Kutta integration. This gives a table of $\frac{\sigma_s}{I_o}$ as a function of $C_\ell(0)$. From this table the value of $C_\ell(0)$ corresponding to the measured value of $\frac{\sigma_s}{I_o}$ is determined using Lagrange-Hermite interpolation formula.

CHAPTER V

RESULTS

A. Probability Density Function for Intensity Scintillation

It has been customary in the literature to test the hypothesis of log-normality of scintillation data by plotting the cumulative probability function of the log-amplitude against a "probability scale" such that if the data is log-normal the resulting curve will be a straight line. The same method can be applied when testing the intensity amplitude for a normal distribution. Since this test would require considerable time if it were applied to every run, it was necessary to use a test that could be incorporated into a computer program in an efficient manner in order to determine which distribution each run more closely fit.

The necessary statistical parameters to make this test were calculated as described in Chapter III and were available on punched data cards. The skewness coefficient was chosen as the parameter to indicate the type of distribution. The skewness coefficient will ideally have zero value for perfectly normal or log-normally distributed data; however, for real data we expected a small value but somewhat greater than zero.

We have chosen the skewness coefficient as the measure of closeness of fit in preference to the chi-square test since the chi-square depends upon the number of class intervals in the sample while the skewness does not. To use the chi-square on a comparative basis would require the generation of a table of chi-square for all possible number of class intervals and would lend to undesirable complexities in the computer program.

In order to use the skewness as a criteria for categorizing the data it was necessary to set bounds on its value. Since the lower bound is zero, it was only necessary to determine the largest value that the skewness could attain which would represent a suitable fit. This was accomplished by plotting several graphs of the cumulative probability for runs with values of skewness ranging from .02 to .79. The chi-square test made on each run was examined to insure that the overall results of the analysis were consistent. A selected number of these graphs are given in Figures 9-17. An inspection of these plots will show that the cumulative probability curve becomes nonlinear with increasing values of skewness. An inspection of many such plots indicated that the curve deviated from linearity much faster when values of skewness became greater than 0.15. For values from 0.0 to 0.15 the curve remained approximately linear.

The results of the statistical analysis of each run was categorized in the following manner: If the skewness for the log-amplitude data is less than or equal to 0.15, the run was categorized as being log-normally distributed. If the value of the skewness meets the above requirement for the intensity data the run was considered to be normally distributed. If both skewness coefficients were greater than 0.15, the run was put in a "neither" category. In addition we have had to include a category for those distributions which were sufficiently close to both a normal and a log-normal distribution that we could not distinguish between them. These runs which have approximately the same skewness for both distributions have been designated as "both".

The results of the categorization for 196 runs are shown in Table I. Only runs with a small number of preprocessing irregularities have

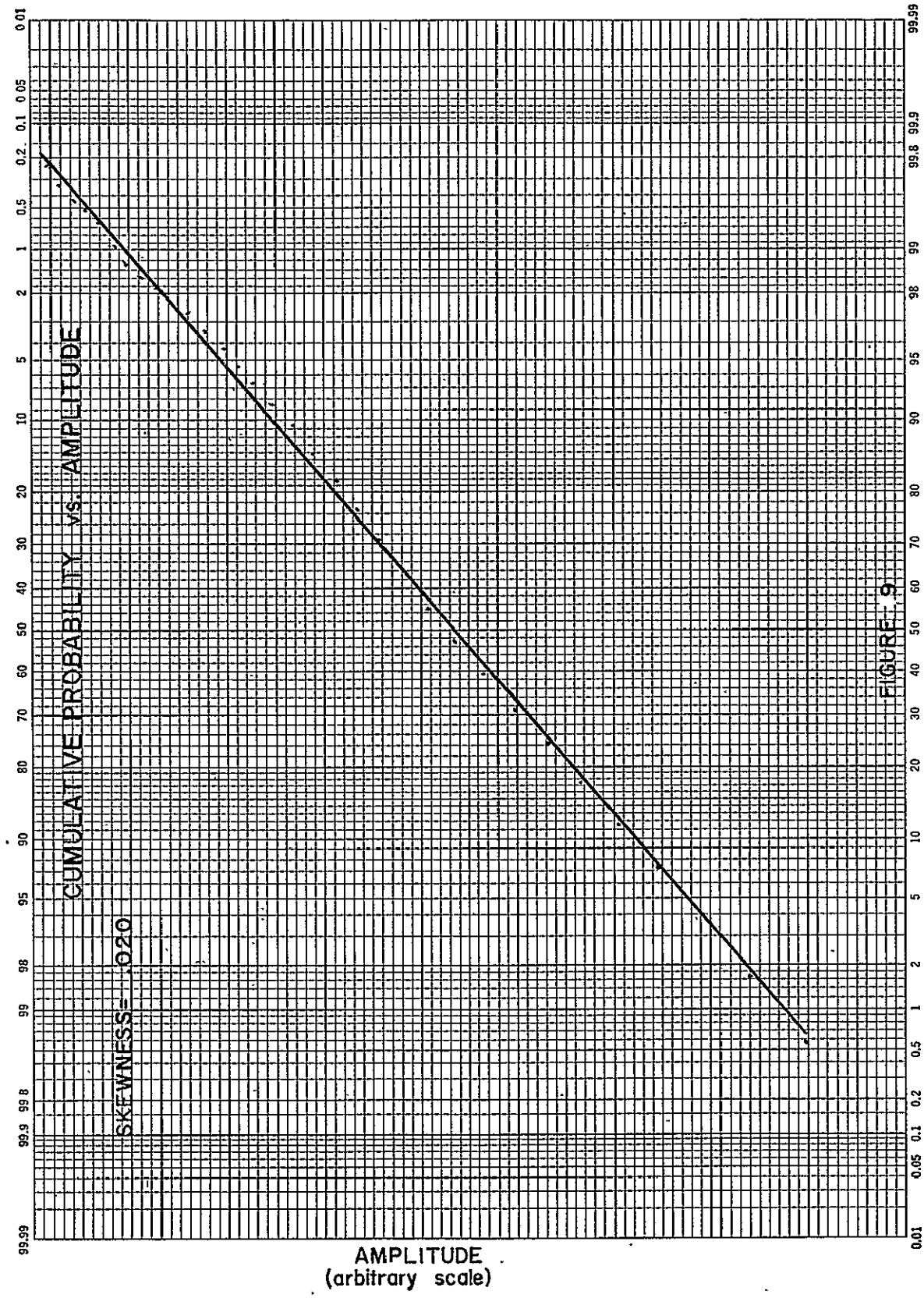

CUMULATIVE PROBABILITY vs. AMPLITUDE
SKEWNESS= .1020
AMPLITUDE
(arbitrary scale)
0.01
0.05
0.1
0.2
0.5
1
2
5
10
20
30
40
50
60
70
80
90
95
98
99
99.8
99.9
99.99

FIGURE 9

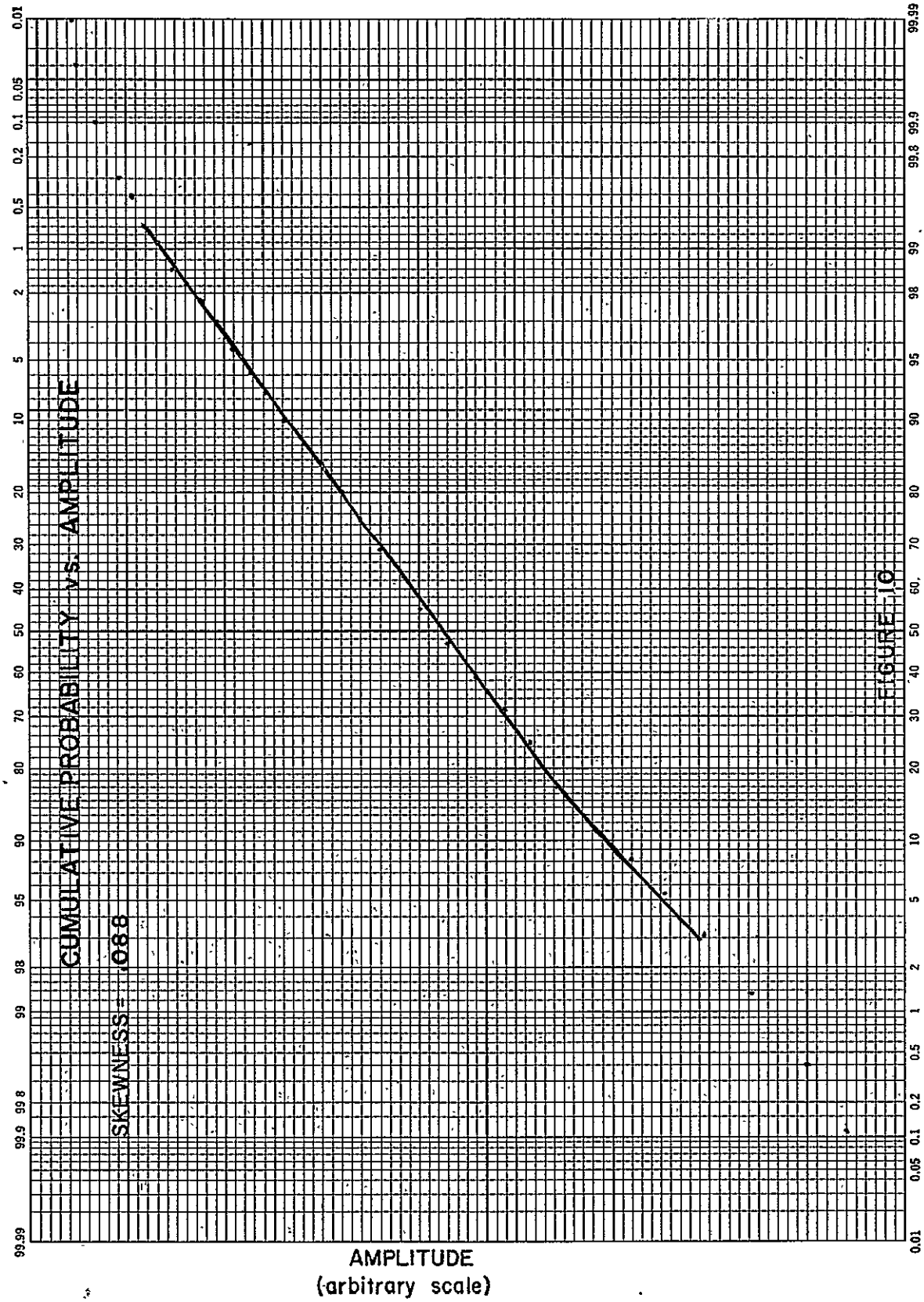

CUMULATIVE PROBABILITY vs. AMPLITUDE
SKEWNESS = .086
0.01 0.05 0.1 0.2 0.5 1 2 5 10 20 30 40 50 60 70 80 90 95 98 99 99.8 99.9 99.99
99.99 99.9 99.8 99 98 95 90 80 70 60 50 40 30 20 10 5 2 1 0.5 0.2 0.1 0.05 0.01
AMPLITUDE
(arbitrary scale)

FIGURE 10

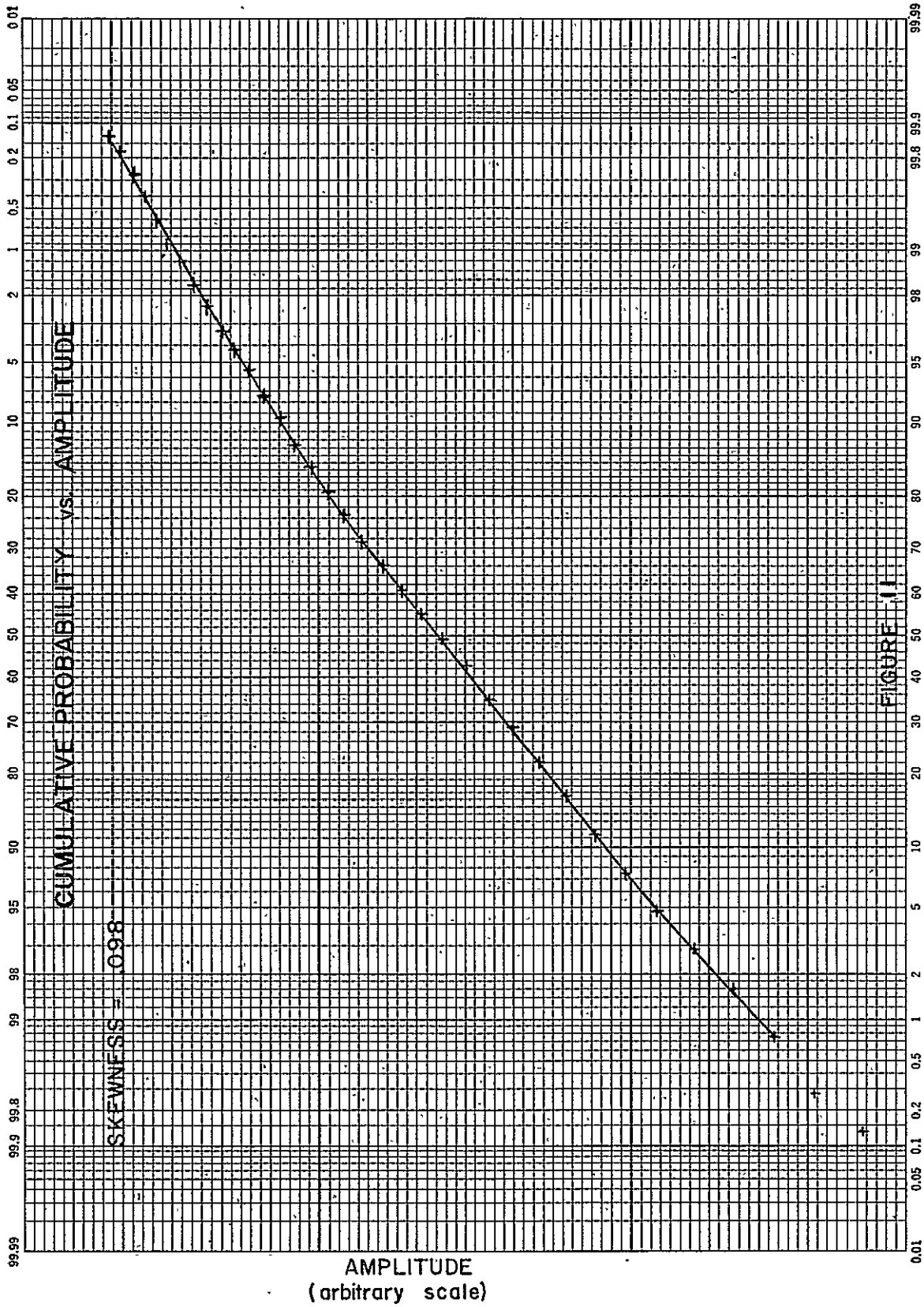

CUMULATIVE PROBABILITY vs. AMPLITUDE
SKEWNESS = -.098
AMPLITUDE
(arbitrary scale)
0.01
0.05
0.1
0.2
0.5
1
2
5
10
20
30
40
50
60
70
80
90
95
98
99
99.8
99.9
99.99

FIGURE 11

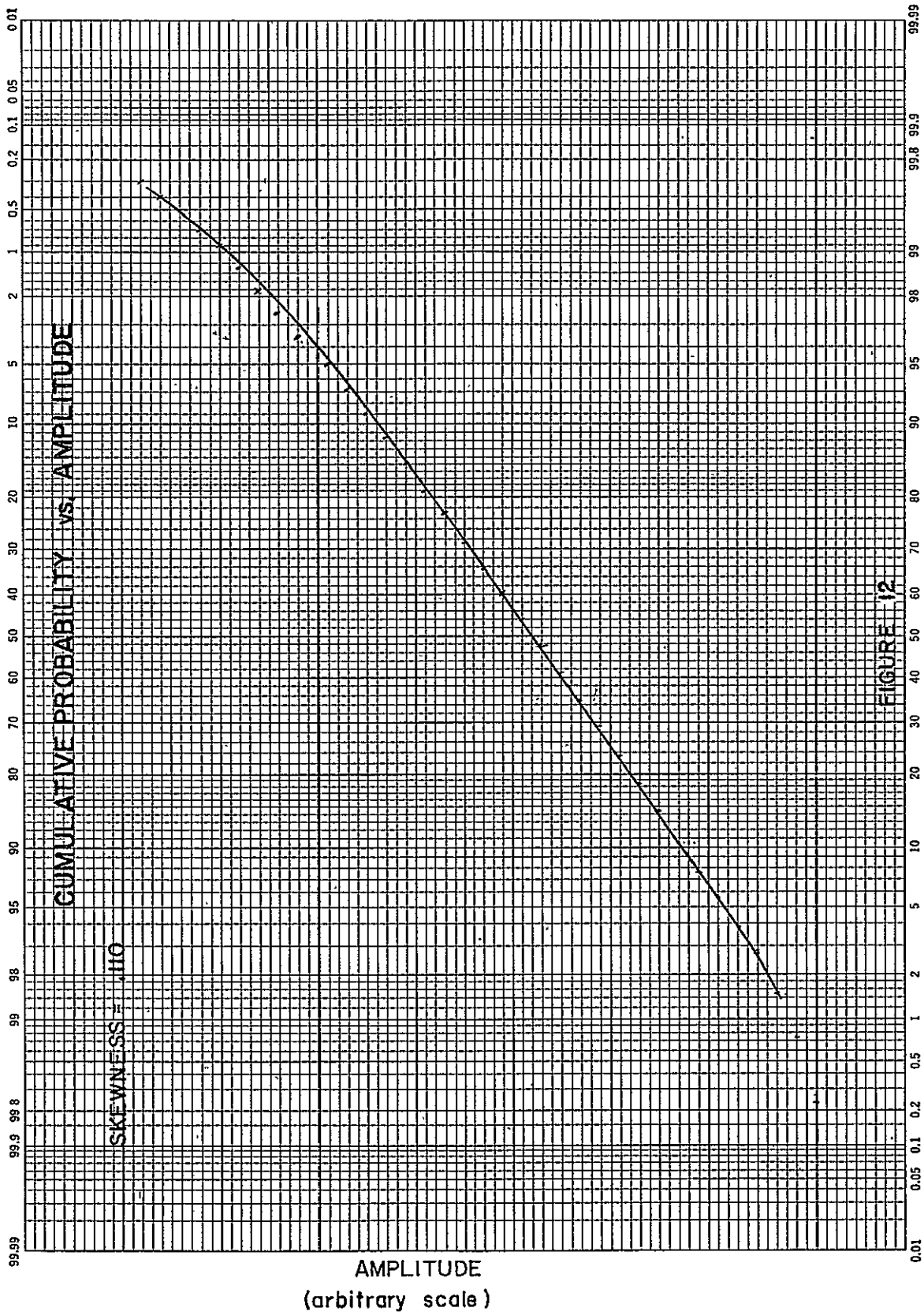

CUMULATIVE PROBABILITY vs. AMPLITUDE
SKEWNESS = .110
0.01 0.05 0.1 0.2 0.5 1 2 5 10 20 30 40 50 60 70 80 90 95 98 99 99.8 99.9 99.99
99.99 99.9 99.8 99 98 95 90 80 70 60 50 40 30 20 10 5 2 1 0.5 0.2 0.1 0.05 0.01
AMPLITUDE
(arbitrary scale)

FIGURE 12

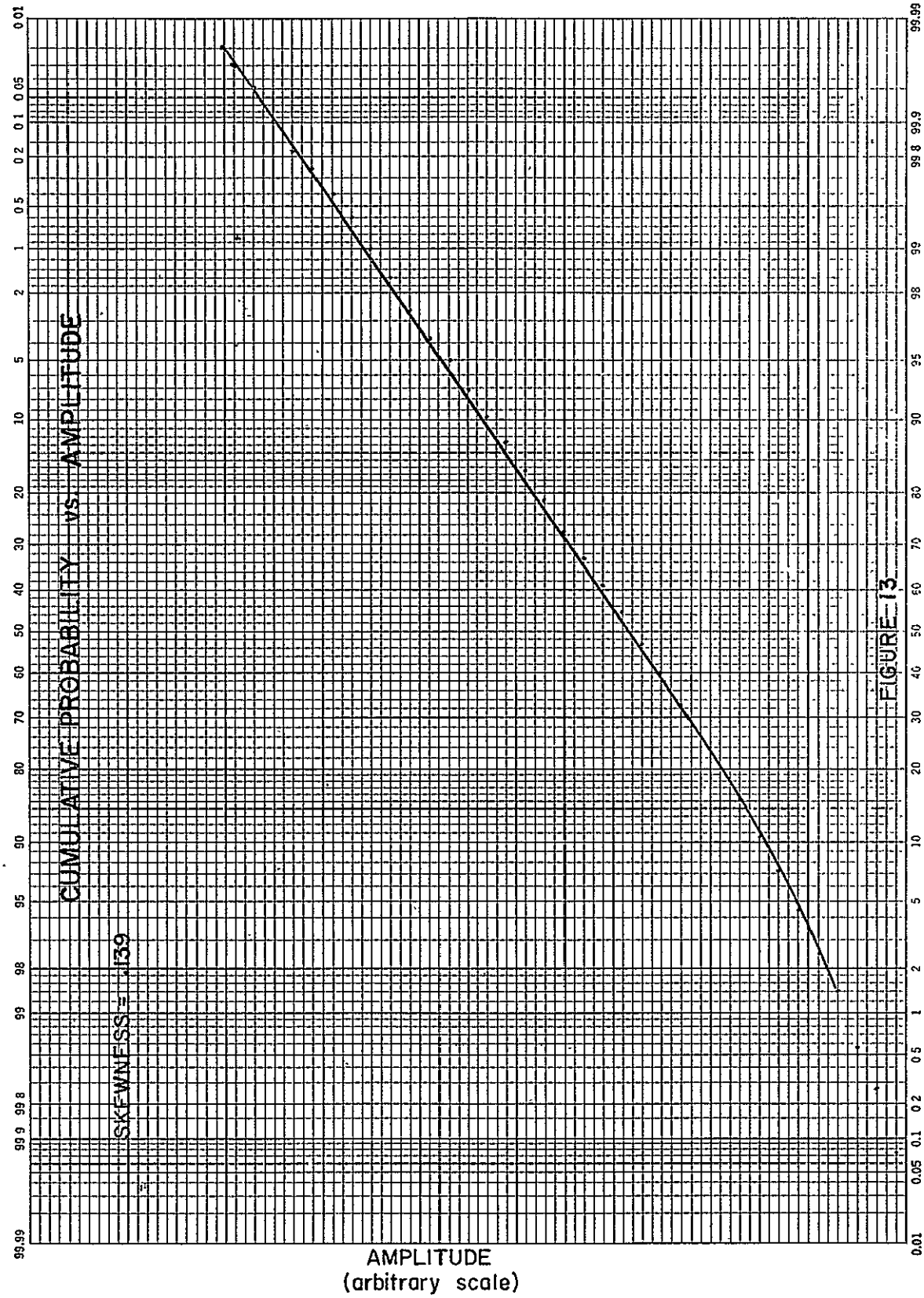

CUMULATIVE PROBABILITY vs. AMPLITUDE
SKEWNESS = .139
AMPLITUDE
(arbitrary scale)
0.01
0.05
0.1
0.2
0.5
1
2
5
10
20
30
40
50
60
70
80
90
95
98
99
99.8
99.9
99.99

FIGURE 13

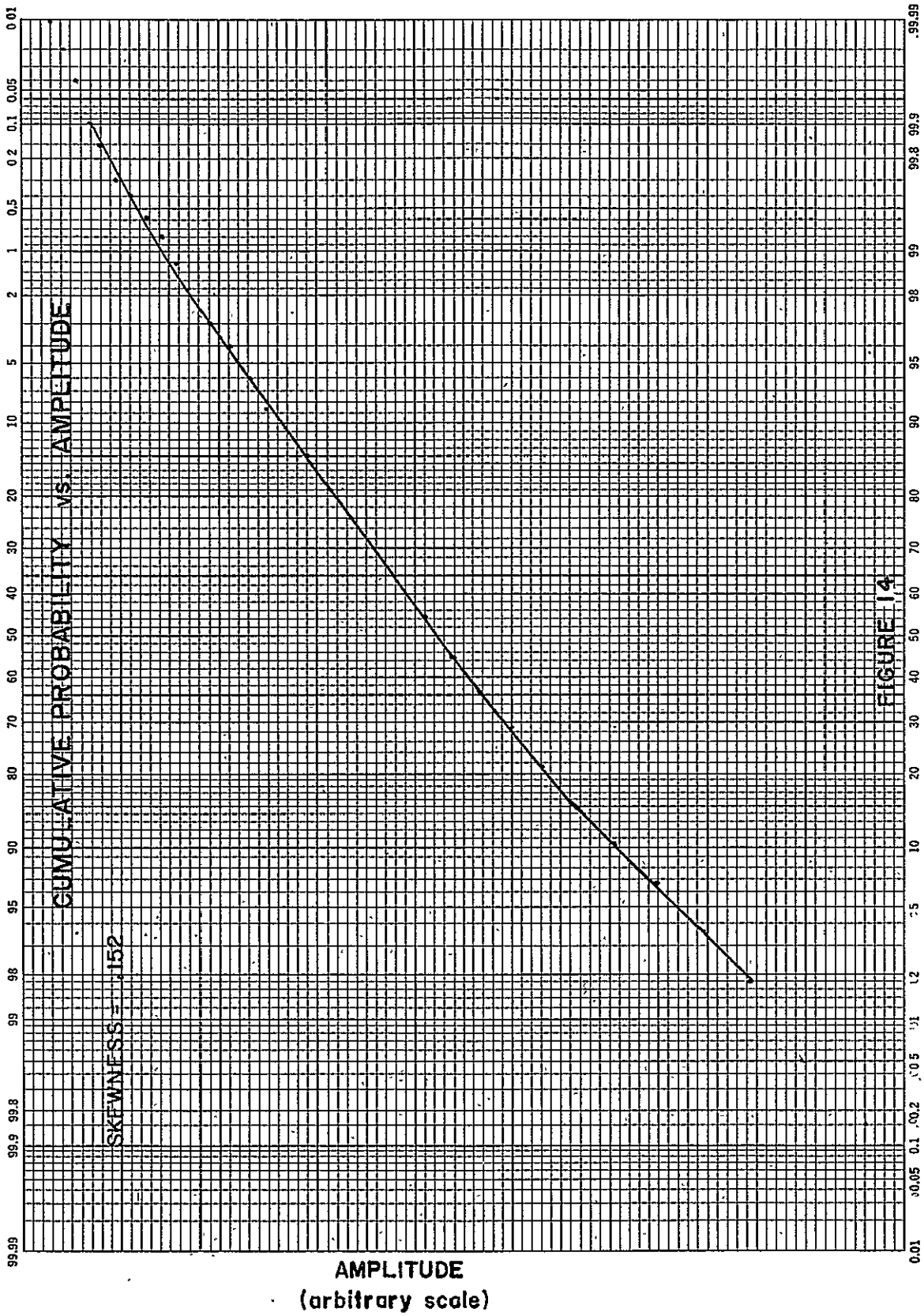
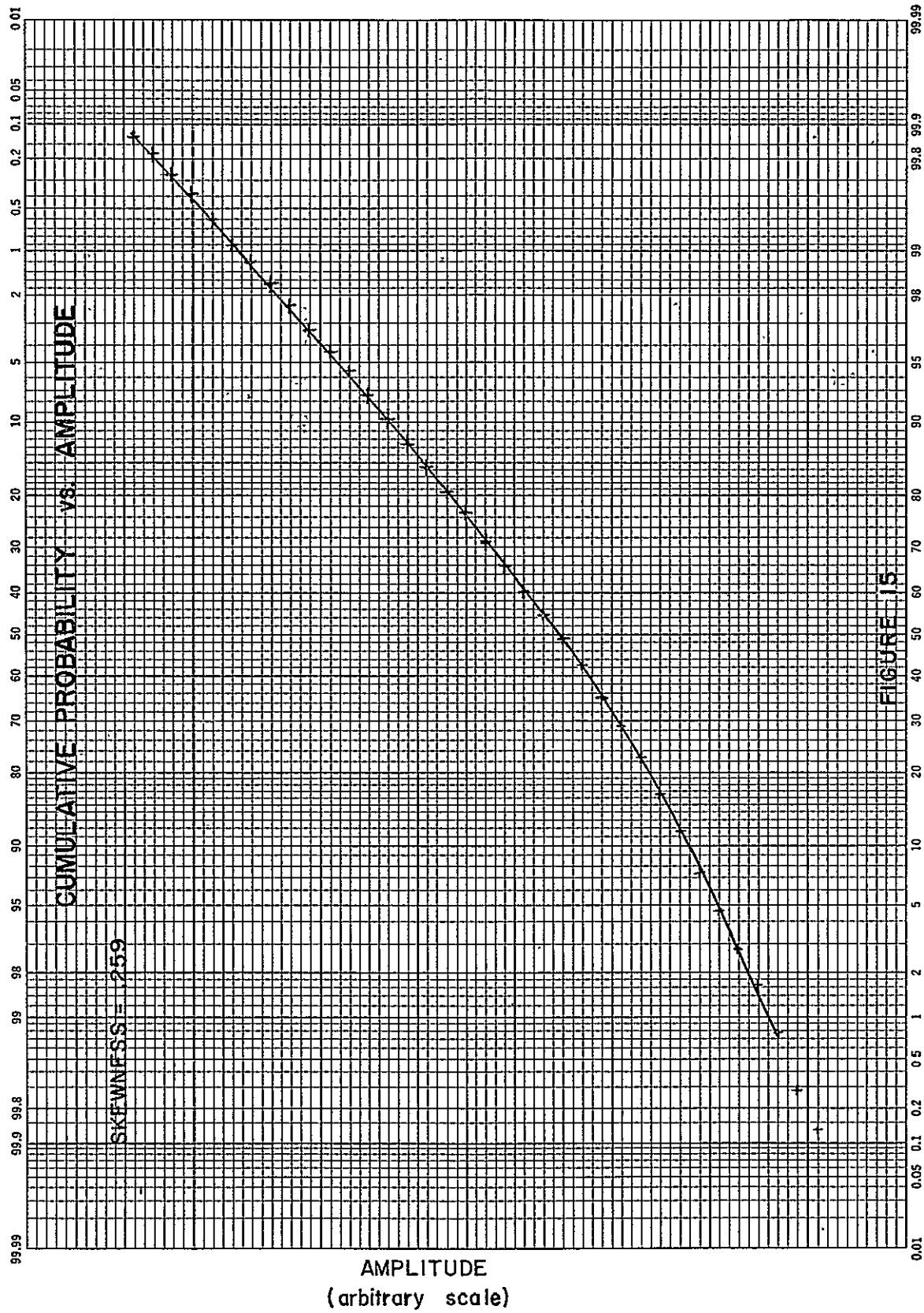

CUMULATIVE PROBABILITY VS. AMPLITUDE
SKEWNESS = .152
AMPLITUDE
(arbitrary scale)

FIGURE 14


CUMULATIVE PROBABILITY vs. AMPLITUDE
SKEWNESS = .259
AMPLITUDE
(arbitrary scale)
FIGURE 15

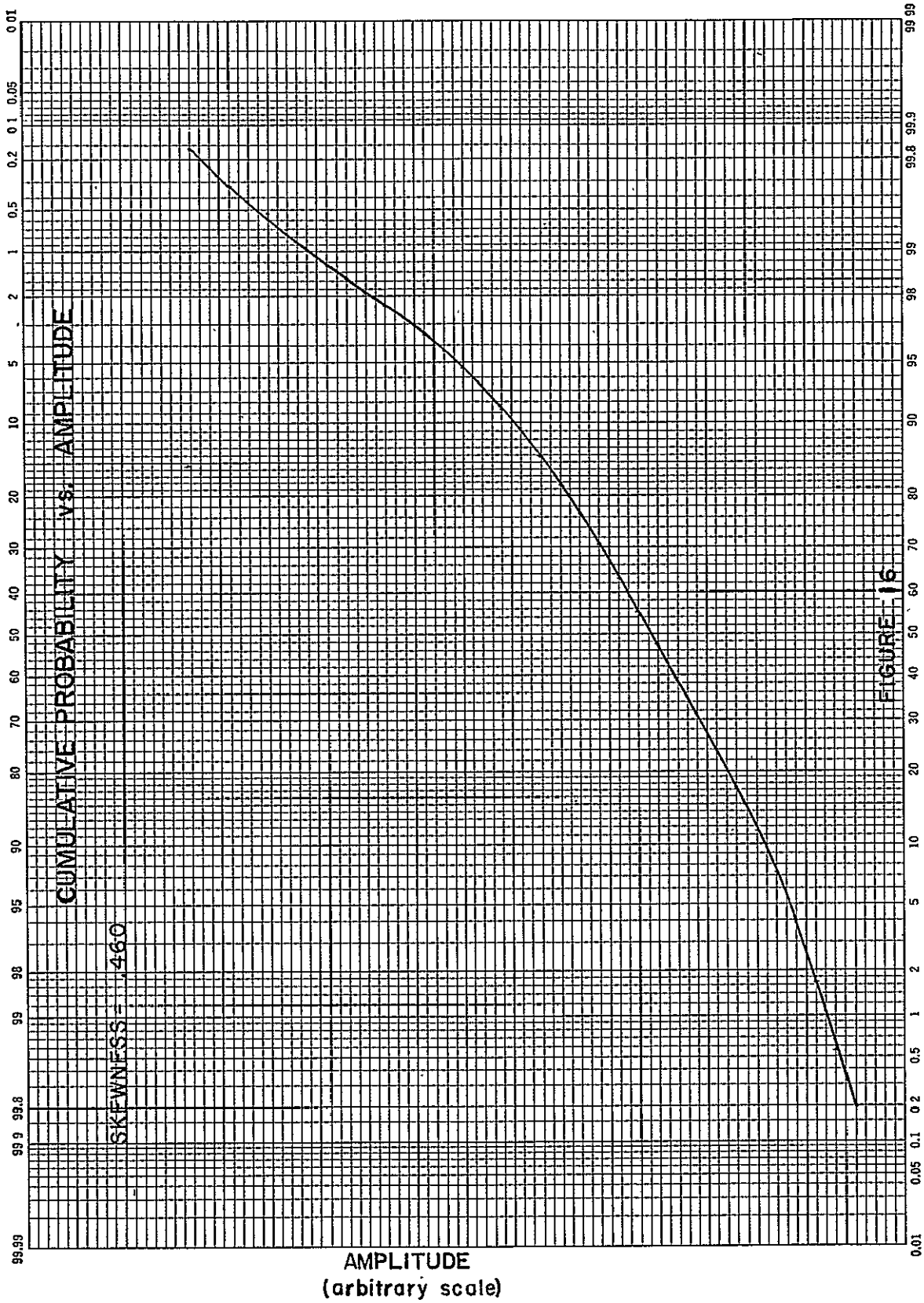

CUMULATIVE PROBABILITY VS. AMPLITUDE
SKEWNESS = .460
AMPLITUDE
(arbitrary scale)
0.01
0.05
0.1
0.2
0.5
1
2
5
10
20
30
40
50
60
70
80
90
95
98
99
99.8
99.9
99.99

FIGURE 16

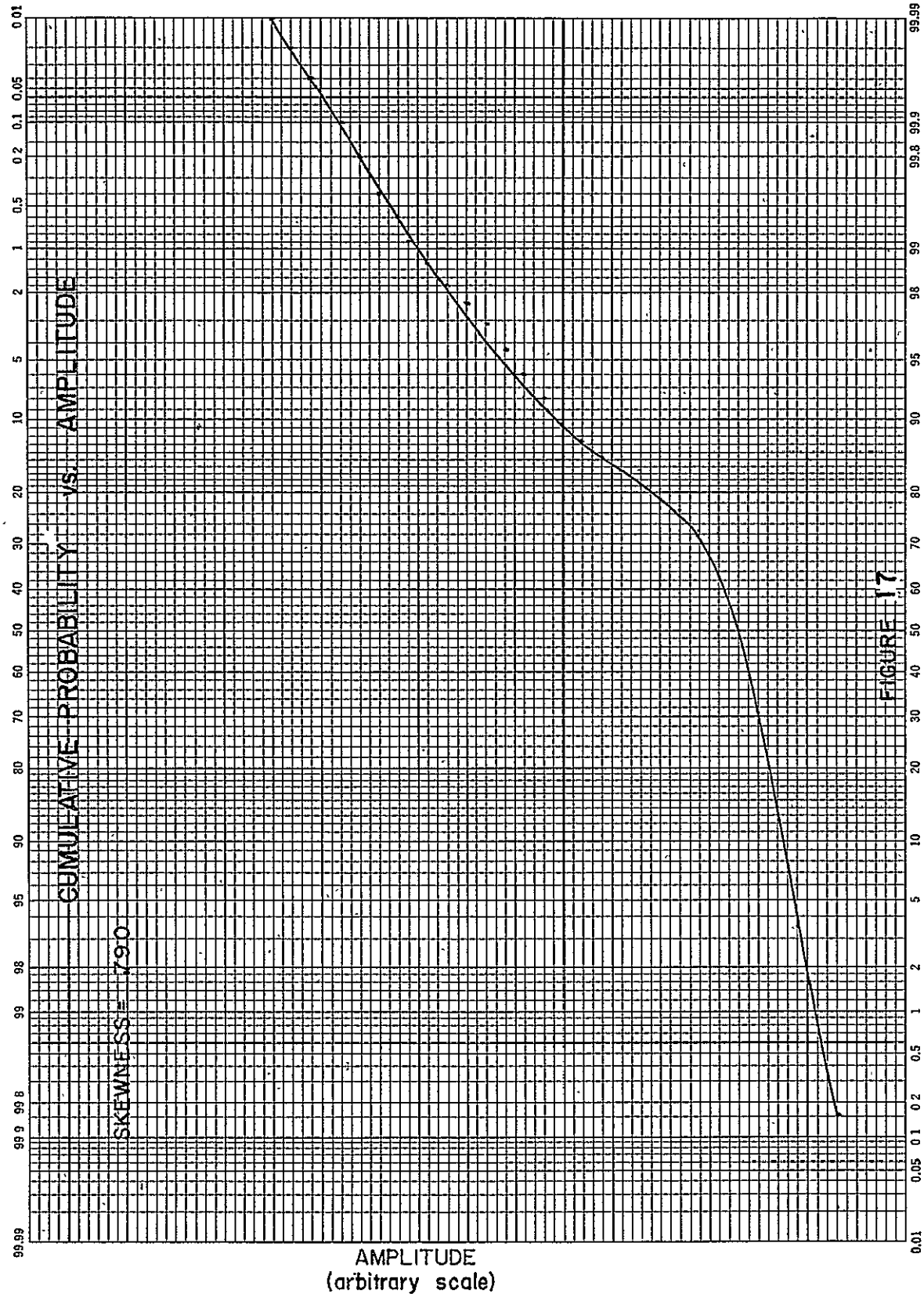

CUMULATIVE PROBABILITY vs. AMPLITUDE
SKEWNESS = 7.90
0.01
0.05
0.1
0.2
0.5
1
2
5
10
20
30
40
50
60
70
80
90
95
98
99
99.8
99.9
99.99
AMPLITUDE
(arbitrary scale)

FIGURE 17

APERTURE SIZE (CM)	NUMBER OF RUNS				
	TOTAL	LOG NORMAL	NORMAL	NEITHER	BOTH
2	4	2	2	0	0
4	17	13	2	1	1
6	30	10	6	13	1
8	43	12	14	11	6
10	102	33	23	37	9

Table 1. Results of Statistical Analysis of Scintillation Measurements.

been tabulated. From this table we see that the two cm. aperture runs were divided equally between normal and log-normal distributions. The results for this aperture size are not considered significant since there were only four runs with small irregularities, and these were taken with a very weak signal. The four cm. runs, which had a reasonably small aperture, yet the recorded signal was strong, show a strong tendency toward log-normality. As the aperture increases the number of runs which differ from log-normal also increases. This behavior may be due to the effect of aperture averaging. Since this process is additive in nature, it would cause the distribution to tend toward a normal curve according to the Central Limit Theorem. This combined with the difficulty in distinguishing normal data from log-normal data could lead to results of the type exhibited by the larger aperture runs.

B. Calculation of Atmospheric Structure Constant

The refractive index structure constant was calculated using equation 4-5 and taking the measured value of the log-amplitude variance $C_\ell(0)$. This equation does not allow for aperture averaging effects. Values of C_n obtained ranged between $1.6 \times 10^{-8}\ M^{1/3}$ and $8.7 \times 10^{-8}\ M^{1/3}$ for a group of runs where the aperture was varied rapidly from two to ten cm. The structure constant computed from two cm. aperture data was usually larger than that computed from ten cm. aperture data by a factor of from two to four. The time required to collect the data for such a group of runs was less than ten minutes. As a comparison, several groups of runs were made in a similar time period holding the aperture constant. The observed variation in the structure constant for these runs was usually about ten percent. This clearly indicates that the observed variation in the structure constant was due to aperture

averaging effects and not to changes in the strength of turbulence between runs.

Using the techniques described in Chapter III, section D, calculations for the structure constant have been refined, allowing for the effects of aperture averaging. For the 212 segments of data which were analyzed, the values of the corrected structure constant ranged from $5.8 \times 10^{-7} M^{1/3}$ to $9.0 \times 10^{-7} M^{1/3}$. These values are characteristic of very strong atmospheric turbulence, which agree with our subjective observations of the scintillation while the data was being taken.

Table II shows four typical sets of runs for various aperture sizes. As can be seen, the variation in the structure constant is significantly reduced when the effects of aperture averaging are included.

It should be noted that the inclusion of aperture averaging effects has a tendency to overcorrect the structure constant variation. This could be an indication of a systematic error in recording the data. Another possibility is that the deep scintillation conditions under which the experimental data was collected produced saturation effects which have not been considered. On the other hand the validity of the basic approach to the problem of aperture averaging in terms of the structure function has been questioned[13]. Therefore it is possible that the aperture correction we have used is not valid. In any case, this method of compensating for aperture averaging effects is a good approximation since the structure constant is far more consistent when corrected than when aperture averaging effects are neglected.

C. Scintillation Frequency Spectrum

The frequency spectrum has been computed using the Fast Fourier

APERTURE SIZE (C M)	RUN 1		RUN 2		RUN 3		RUN 4	
	UNCOR C N	COR C N	UNCOR C N	COR C N	UNCOR C N	COR C N	UNCOR C N	COR C N
10	.878	7.50	.178	5.88	.179	5.88	.885	7.47
8	.686	7.50	.217	6.40	.241	6.48	.709	7.53
6	.547	7.61	.326	7.15	.425	7.35	.635	7.73
4	.563	8.05	.451	7.90	.617	8.10	.590	8.09
2			.550	8.77	.590	8.82	.565	8.79
MEAN	.662	7.67	.344	7.22	.410	7.33	679	7.92
%VARIATION	51.7	12.9	109	40	100	40.2	47.2	16.7

$$(CN) \text{ IN } (\text{METERS})^{1/3} \times 10^{-7}$$

Table 2. Comparison of Structure Constant Corrected for Aperture Averaging Effects with its Uncorrected Value.

Transform techniques described above. This calculation was performed only on selected data runs due to the time required for such a calculation. Also the computed spectra were very consistent so it was felt that analysis of additional runs would yield little additional information.

The computed spectra cover a range of frequencies from DC to 90 Hz, the upper limit being set by the 180 sample per second sampling rate. A typical spectrum is shown in Figure 18. Although the computer plotted frequency components out to 90 Hz only components out to 20 Hz are shown in the figure for the sake of clarity. Above 20 Hz the spectrum continued to decrease linearly so that no appreciable frequency components above 40 Hz were observed.

D. Heterodyne Detection

The effect of atmospheric turbulence on the performance of the equipment operating as a heterodyne communication system was investigated. This was accomplished by recording the amplitude of the 10 Mhz heterodyne signal at the output of the I. F. amplifier. Also the transmitter was modulated with a 1 Khz signal which was recorded at the output of the receiver's F. M. descriminator. It was found that neither signal showed any effect attributable to atmospheric turbulence large enough to be accurately measured. Even under conditions of deep scintillation encountered during the course of this experiment, the atmospherically induced noise was of the same magnitude or smaller than system noise. It was also found that clear voice communications were possible over this range under the worst conditions of scintillation encountered.

While these results clearly indicate the feasibility of using a

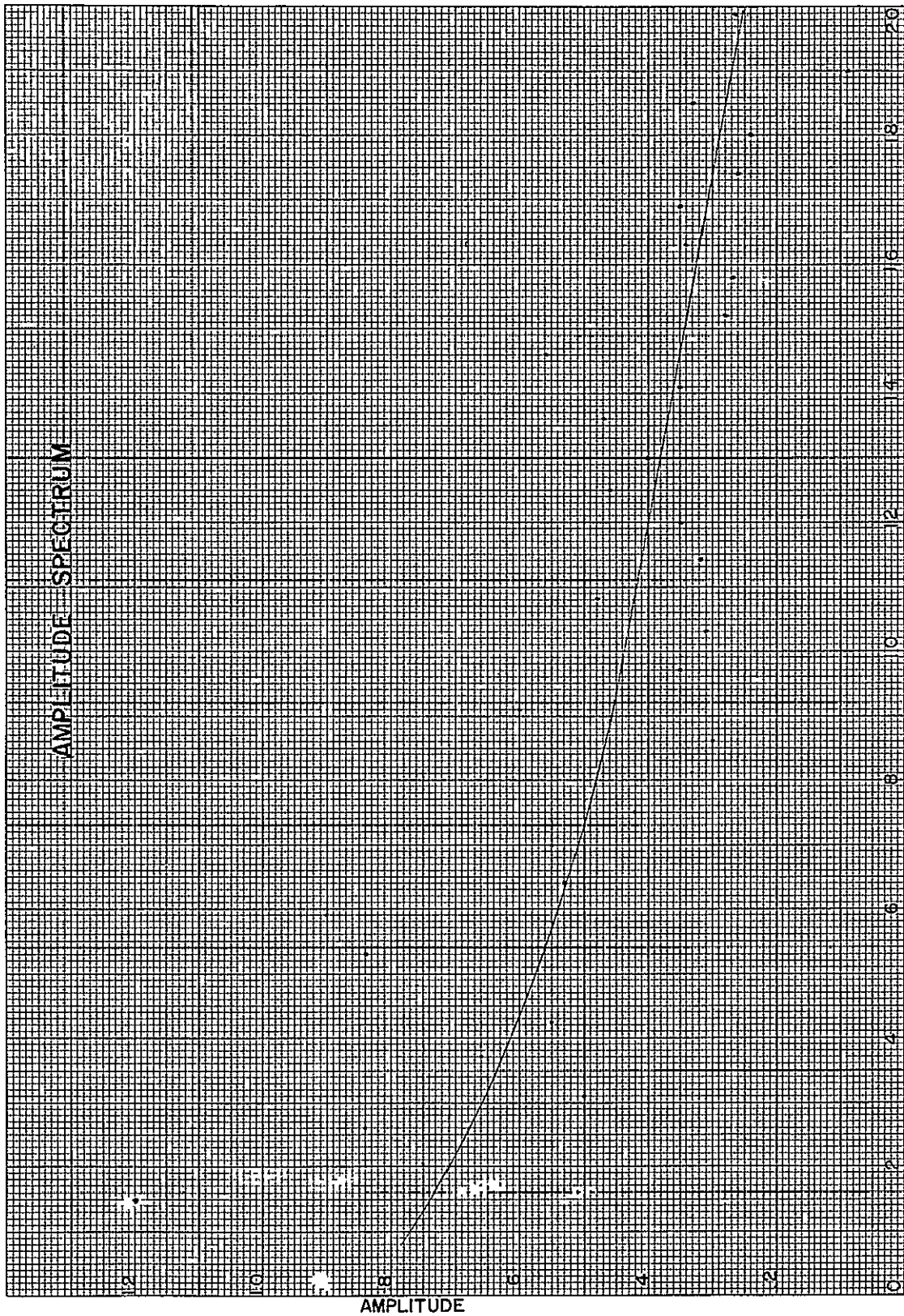

AMPLITUDE SPECTRUM
FREQUENCY Hz
AMPLITUDE
0
2
4
6
8
10
12
14
16
18
20
.2
.4
.6
.8
1.2

FIGURE 18

heterodyne CO_2 laser system to communicate through the earth atmosphere, they were somewhat disappointing since they did not permit a quantitative measurement of noise induced by atmospheric turbulence.

CHAPTER VI

SUMMARY AND CONCLUSION

The results of the scintillation measurements made on the 10.6 micron wavelength laser beam tend to confirm the log-normal model for small receiver apertures. The data for the larger apertures did not seem to fit the log-normal or normal models with any consistency. One possible explanation for this could be aperture averaging. It is possible that the larger aperture sizes were not large enough with respect to the correlation distance to cause the distribution to be normal, but yet large enough to cause the distribution to deviate from log-normally. This in addition to the difficulty in distinguishing between the two distributions could have caused the results to be inconclusive.

The value of the refractive index structure constant computed was found to lie within the range of values for this constant as calculated by Fried. The value of this constant was found to decrease with increasing aperture size. Equations developed by Fried were used to correct the structure constant for aperture effects. This technique seemed to give a slightly larger value of the aperture averaging effect than was observed. Although this may indicate an inaccuracy in the theoretical expression, a systematic error in the experimental measurements cannot be firmly ruled out. These calculations were significant in that they indicated in a quantitative manner the nature of aperture averaging.

The spectral analysis indicated that low frequency components of the scintillation were predominant. The magnitude of the scintillation decreases linearly with increasing frequency. Above 20 Hz the

scintillation is negligible.

The feasibility of optical heterodyne communication at 10 microns through extreme turbulence was demonstrated. To our knowledge, this was the first system of this kind to be operated through the atmosphere at this path length. We feel that the successful operation of the communications system under extreme scintillation conditions was a significant result.

It would be a great advantage in this type experiment to reduce the data immediately after it is taken. Information gained from the speedy reduction should give the experimenter a knowledge of how the system is performing and aid in making better measurements.

As a result of performing this experiment and surveying the results of other investigations, it is clear that the atmospheric problem is far from being completely solved. The log-normal model needs to be further verified for other wave lengths. The variance and the structure constant should be investigated under as wide a variety of weather conditions as possible and should be correlated to the variations of the meterological parameters.

After having carried out this type experiment, the need for several refinements in the procedure was realized. Before any data is taken, a thorough analysis of system noise should be made. Sensitive noise measuring instruments should be employed. The noise of the transmitting laser should be recorded simultaneously with scintillation measurements. The background light effect should be further studied and perhaps a method other than signal chopping used to handle the problem. Mechanical stability of equipment is a problem that needs investigation since even very small vibrations could cause the beam to shift out of alignment

with the receiver.

Aperture averaging effects need to be investigated with many different size apertures for all wave lengths. The relationship between correlation distance and aperture size needs to be determined. The determination of correlation distance in itself would be an interesting experiment.

APPENDIX A

Introduction

This appendix further describes the computer program employed to reduce the atmospheric data. The program is written in Fortran IV language for the IBM-360 Model 50 computer at the University of Alabama. The program operates in the following manner: The MAIN or supervisory routine accepts instructional and operating data for the program. It reads the atmospheric data from the magnetic tape and calls subroutine EXT to extract the intensity signal. EXT calls on subroutines LIMIT, FILL, HIST, and AMPX to perform the extration. When a file of data has been processed MAIN calls subroutine PRINT to print a table of the irregularities that occurred during extraction. MAIN then calls subroutine STAT and/or FFT to perform the statistical and spectral analysis. STAT calls on subroutine CHI to perform the chi-square test which in turn calls on a Simpsons rule integration subroutine SIMP. FFT calls on two package subroutines FOURT and PLOT which perform and plot the spectral analysis.

Routine Main

Routine MAIN's first step is to read the identification record from the magnetic tape. This is done by calling subroutine RID. RID actually does the reading and stores the data in a common array to be printed by MAIN in the next step. The numerical instructions and operating constants are then read in as data on punched cards. These are read in as variable names and are used in the various subroutines and are discussed as part of the description of these subroutines.

Processing actually begins when MAIN reads in 4 additional instructional constants on punched cards. These are given according to their variable name as

NAME	FUNCTION
FILES	The number of the data file to be processed.
ISTART	The record number within the file from which data will start being taken.
ICHNO	The channel number of the magnetic tape to be processed.
IRCEND	The record number at which processing is to stop.

The values of these numerical constants enable the user to process any record or segment of records within a data file on any of the five channels. Each time a new file is to be processed these variables must be read in for the new file. The read statement for these variables is in a loop so that when processing of a file is completed the program returns to this statement to get instructions for processing the next file. After the last file has been processed the program is stopped by entering a negative number for the variable FILES.

The program now moves the tape to the desired file and record by calling subroutine REDREC. When the first record to be processed is found, it is stored in a common array IBUF. MAIN then transfers the data in IBUF into a work array P. REDREC is called again to read the next record which is transferred by MAIN into an auxilary array AUX. This is done in order to have the next record available in an array since it is sometimes necessary for the program to "look" into the following record before processing in a record is completed.

The record of data in array P is now processed by calling sub-

```
FORTRAN IV G LEVEL 1, MOD 4            MAIN            DATE = 70122          02/35/57

0001                INTEGER FILES,FILCK
0002                COMMON P(1000),AUX(1000),INDO(10),NCNT(300),JOVF,JUNF,
                   *BASE(2,10),SIG(10),IBLOC(2,10),KK,MM,XL90,XL10
                   *, DUMMY(2700)
0003                COMMON/AAA/IPRINT
0004                COMMON/AP/AMP(10000),NDATA
0005                COMMON/RLC/IRTN
0006                COMMON/RAT/IRCEND
0007                COMMON/GO/N1,N3,N4,FM,IDO
0008                COMMON/NPTS8/N8
0009                COMMON/PR/IE(7,300)
0010                COMMON/UUU/NPNCH
0011                DIMENSION IBUF(1000)
0012                DIMENSION IA(3),IB(5)
0013                CALL RID(I1,I2,IB)
0014                IF(I1) 155,150,155
0015             155  PRINT 157,FILCK
0016             157  FORMAT(' ***  END OF FILE  ***',I8)
0017              150 IF(I2) 158,161,158
0018             158  PRINT 159
0019             159  FORMAT('  ***  READ ERROR  ***')
0020              161 CONTINUE
0021                  NTAPE=IB(1)
0022                  PRINT 160,IB
0023             160  FORMAT('  TAPE NUMBER=',1A4,
                   1'  IDCAL=',1A4,
                   1'  BASE=',1A4,
                   1'  NUMBER OF CHANNELS=',1A4,
                   1'  NUMBER OF SCANS=',1A4)
0024                  FILCK=1
0025                  READ(5,9921) FMAX,DT,FM,N1,N3,N4,IDO,ISSB
0026            9921  FORMAT(3E10.0,5I10)
0027                  READ(5,3350)IPRINT,NPNCH
0028             3350 FORMAT(2I10)
                C     READ INSTRUCTIONS AND TOLERANCES
0029                  READ(5,360) IBTYP,IFLAG,NCI,NOSCAN,NOCHAN,L1,L2
0030             360  FORMAT(7I10)
0031                  READ(5,1991) TOL1,TOL2,TOL3
0032             1991 FORMAT(3E10.0)
0033                  PRINT 361,IBTYP,IFLAG,NCI,NOSCAN,NOCHAN,TOL1,TOL2,TOL3,L1,L2
0034             361  FORMAT('  TYPE BASE CALCULATION- **********************************
                   ******  ***** IBTYP     *****     'I10/,
                   *'  TYPE STATISTICS CALLED FOR  **********************************
                   ****** IFLAG     *****     ',I10/,
                   *'  NUMBER OF CLASS INTERVALS  **********************************
                   ****** NCI       *****     ',I10/,
                   *'  NUMBER OF SCANS PER RECORD  **********************************
                   ****** NOSCAN    *****     ',I10/,
                   *'  NUMBER OF CHANNELS ON TAPE  **********************************
                   ****** NOCHAN    *****     ',I10/,
                   *'  TOLERANCE ON BASE LIMITS  **********************************
                   ****** TOL1      *****     ',F10.3/,
                   *'  TOLERANCE ON SIGNAL LIMITS  **********************************
                   ****** TOL2      *****     ',F10.3/,
                   *'  NUMBER OF SIGNAL AND BASE POINTS REQUIRED PER CYCLE  ********
                   ****** TOL3      *****     ',F10.3/,
```

```
FORTRAN IV G LEVEL 1, MOD 4            MAIN              DATE = 70122          02/35/57

                *'  NUMBER OF PASSES REQUIRED TO ABORT CYCLE  ******************
                ****** L1        *****     ',I10/,
                *'  NUMBER OF BASE POINTS REQUIRED FOR SIGNAL POINT SEARCH  *****
                ****** L2        *****     ',I10)
0035             PRINT 9111,NPNCH,IPRINT,ISSB,FMAX,DT,FM,N1,N3,N4,IDO
0036           9111 FORMAT('  PUNCHED OUTPUT FLAG  ********************************
                ******  ***** NPNCH     *****     ',I10/,
                *'  PRINT ERROR TABLE FLAG  ************************************
                ****** IPRINT    *****     ',I10/,
                *'  TYPE ANALYSIS ROUTINE WANTED  ******************************
                ****** ISSB      *****     ',I10/,
                *'  INFORMATION FOR FFT ROUTINE  *******************************
                ****** FMAX      *****     ',F10.3/,65X,'***** DT         *****
                *',F10.3/,65X,'***** FM        *****     ',F10.3/,65X,'***** N1
                *    *****     ',I10/,65X,'***** N3        *****     ',I10/,65X,'**
                **** N4        *****     ',I10/,65X,'***** IDO       *****     ',I1
                *0/)
              C     READ DATA FOR TAPE PROCESSING
0037             LFILE=0
0038           511 READ(5,882)FILES,ISTART,ICHNO,IRCEND
0039             N8=0
0040             NDATA=0.0
0041             DO 8892 LJ=1,7
0042             DO 8892 JI=1,300
0043           8892 IE(IJ,JI)=0
0044             IRECNT=ISTART-1
0045             IEXIT=0
0046             IF(FILES-LFILE)22,22,987
0047           987 IREC=0
0048           22  CONTINUE
0049             LFILE=FILES
0050           882 FORMAT(4I3)
0051             PRINT 901,FILES,ICHNO,ISTART,IRCEND
0052           901 FORMAT(1H1,'     PROCESSING FILE **********',I5/,
                *'     CHANNEL ****************************',I5/,
                *'     BEGIN AT RECORD ********************',I5/,
                *'     STOP PROCESSING AT RECORD ***********',I5)
0053             IRTN=1
0054             IF(FILES)991,9,9
0055           991 PRINT 992
0056           992 FORMAT(10X,'******  PROGRAM STOP *****')
0057             STOP
0058           9   CALL REDREC(FILES,IREC,FILCK,IBUF,TIME,N,NOSCAN,NOCHAN,ICHNO)
              C     MOVE TAPE TO DESIRED FILE
0059             IF(FILCK-FILES)9,887,887
              C     READ RECORD, STORE RECORD IN ARRAY IBUF
0060           887 CALL REDREC(FILES,IREC,FILCK,IBUF,TIME,N,NOSCAN,NOCHAN,ICHNO)
              C     MOVE TAPE TO DESIRED RECORD
0061             IF(IREC-ISTART)887,889,889
              C     STORE CONTENTS OF IBUF INTO MAIN ARRAY P
0062           889 DO 888 L=1,N
0063           888 P(L)=IBUF(L)
              C     READ NEXT RECORD INTO IBUF
0064             CALL RED REC(FILES,IREC,FILCK,IBUF,TIME,N,NOSCAN,NOCHAN,ICHNO)
              C     STORE CONTENTS OF IBUF INTO AUXILIARY ARRAY AUX
0065             DO 500 L=1,N
```

```
FORTRAN IV G LEVEL 1, MOD 4            MAIN              DATE = 70122          02/35/57

0066          500  AUX(L)=IBUF(L)
0067               GO TO 501
0068          502  DO 503 L=1,N
0069          503  P(L)=AUX(L)
0070               CALL RED REC(FILES,IREC,FILCK,IBUF,TIME,N,NOSCAN,NOCHAN,ICHNO)
             C     STORE IBUF INTO AUX
0071               DO 504 L=1,N
0072          504  AUX(L)=IBUF(L)
0073          501  CALL EXT(N,L1,L2,IRECNT,TOL1,TOL2,TOL3,NCI,IBTYP)
             C     STOP PROCESSING ON DESIRED RECORD
0074               IF(IEXIT) 1000,1000,507
0075         1000  IF(NDATA.EQ.10000) GO TO 507
0076          661  IF(IREC-IRCEND) 502,221,221
0077          221  DO 2293 I=1,N
0078          2293 P(I)=AUX(I)
0079               IEXIT=1
0080               GO TO 501
             C     PRINT ERROR TABLE
0081            507 CALL PRINT(1)
0082               IF(ISSB.EQ.2) GO TO 2514
0083                     CALL FFT(N,DT,FMAX)
0084               IF(ISSB.EQ.1) GO TO 511
0085          2514 CALL STAT(NTAPE.ICHNO.FILES.NCI.IFLAG)
0086               GO TO 511
0087               END
```

routine EXT. When EXT completes the processing it returns program control to MAIN. The program checks the variable IEXIT to see if the record just processed is the last record in the file. If it is not, the program checks to see if the next record is the last. If this record is not the last, the program calls in the next record and continues processing. However, if it is the last record in the file to be processed, the program transfers the contents of the AUX array (which contains the last record) into work array P. IEXIT is set to 1 and EXT called to process the last record. When EXT returns program control to MAIN, IEXIT indicates that processing of this file has been completed.

The program then calls subroutine PRINT to print the irregularity table. The analysis of the extracted signal continues by calling the statistical analysis subroutine STAT or the spectral analysis subroutine FFT. When the analysis is completed for this file of data, control is returned to MAIN which loops back to read the instructions for the next file to be processed.

Subroutines

A description of the subroutines called in the program is given in this section. The only subroutines not listed are FOURT and PLOT, since they were used in a package furnished by the computer department. A Fortran list is included after the description of each subroutine.

SUBROUTINE RID (13, 14, IC)

This subroutine reads the identification file which is the first file on the tape. This routine uses 3 subroutines that are especially written for the University of Alabama IBM-360 Model 50 computer. The first, NTRAN, actually reads the tape and stores the data into an array.

The data is then transferred from this array and decoded for system compatibility using utility subroutines MOVE and TRNSL. This routine would probably require modification or rewriting if it were used on another machine.

SUBROUTINE REDREC (FILES, IREC, FLICK, IBUF, TIME, N, NOSCAN, NOCHNO, ICHNO)

Subroutine REDREC is called by MAIN to read and reformat the data from the magnetic tape. REDREC calls on the special utility subroutine, NTRAN to read the tape. Utility subroutines MOVE and TRNSL are called to convert the 7 track tape output into the byte system. REDREC must unpack from the multiplexed data the desired channel and convert the binary code to conventional base ten numbers. It must also keep up with the record and file number that it is reading. Provisions were made in REDREC to indicate read errors that might occur in NTRAN. This subroutine would probably require modification if used on another machine.

Argument Variables

FILES - Number of files to be processed.

IREC - Record number as counted by REDREC.

FLICK - File number as counted by REDREC.

IBUF - Array containing raw data.

TIME - Not used.

N - Number of data points per record (either 200 or 1000).

NOSCAN - Number of scans per record per channel (either 200 or 1000).

NOCHAN - Number of channels multiplexed on the tape.

ICHNO - Channel number to be processed.

```
FORTRAN IV G LEVEL 1, MOD 4            RID                DATE = 70122          02/35/57

0001                SUBROUTINE RID(I3,I4,IC)
0002                INTEGER IB(5),IA(3), BUF(501),FLCNT,TTB(3),BLK,FILES,FILCK,
                   *IBUF(1),IC(1)
0003                M=1
0004                I1=0
0005                I2=0
0006           5    CALL NTRAN(1,2,3,IA,K,2,-2004,BUF,L,22)
0007           2    IF(K+1) 1,2,3
0008           3    CALL MOVE(IB,1,IA,1,4)
0009                CALL TRNSL(IB,4,TTB)
0010                DATA TTB/'0123456789'/
0011                DATA BLK /' '/
0012                IB(2)=BLK
0013                CALL MOVE(IB(2),1,IA,5,1)
0014                CALL TRNSL(IB(2),1,TTB)
0015                IB(3)=BLK
0016                CALL MOVE(IB(3),1,IA,6,1)
0017                CALL TRNSL(IB(3),1,TTB)
0018                IB(4)=BLK
0019                CALL MOVE(IB(4),1,IA,7,2)
0020                CALL TRNSL(IB(4),2,TTB)
0021                CALL MOVE(IB(5),1,IA,9,4)
0022                CALL TRNSL(IB(5),4,TTB)
0023                GO TO (6,7), M
0024           6    DO 14 I=1,5
0025          14    IC(I)=IB(I)
0026                I3=I1
0027                I4=I2
0028                RETURN
0029           1    IF(K.EQ.-2) GO TO 4
0030                I2=1
0031                CALL NTRAN(1,22)
0032                GO TO (6,7),M
0033           4    I1=1
0034                CALL NTRAN(1,22)
0035                GO TO (6,7),M
0036                ENTRY REDREC(FILES,IREC,FILCK,IBUF,TIME,N,NOSCAN,NOCHAN,ICHNO)
0037                M=2
0038          13    IREC=IREC+1
0039                CALL NTRAN(1,22)
0040           9    IF(L+1)8,9,10
0041          10    TIME=BUF(1)
0042                N=(L-4)/(2*NOCHAN)
0043                K=3+2*ICHNO
0044                DO 11 I=1,N
0045                IBUF(I)=0
0046                J=0
0047                CALL MOVE(J,4,BUF(1),K,1)
0048                CALL MOVE(IBUF(I),4,BUF(1),K+1,1)
0049                IBUF(I)=IBUF(I) + 64*J
0050                IF(IBUF(I).GE.1024) IBUF(I)=1024-IBUF(I)
0051          11    K=K+2*NOCHAN
0052                CALL NTRAN(1,2,-2004,BUF,L)
0053                RETURN
0054           8    IF(L.EQ.-2) GO TO 12
0055                PRINT 100
```

```
FORTRAN IV G LEVEL 1, MOD 4            RID                DATE = 70122      02/35/57

0056          100  FORMAT('0******  READ ERROR  ******')
0057               CALL NTRAN(1,22)
0058               CALL NTRAN(1,2,-2004,BUF,L)
0059               GO TO 13
0060          12   CONTINUE
0061               FILCK=FILCK+1
0062               CALL NTRAN(1,22)
0063               GO TO 5
0064          7    CONTINUE
0065               IREC=0
0066               GO TO 13
0067               END
```

SUBROUTINE EXT (NK, L1, L2, IREC, TOL1, TOL2, TOL3, NCI, IBTYP)

This subroutine is by far the most complex routine in the program. It extracts the intensity amplitudes from the chopped data. A written explanation of EXT will not be given due to its complexity. Included instead is a flow diagram. It is hoped that the interested reader can use the description given in the text along with the flow chart to understand the operation of this routine. As an additional aid, the important variable names are given a brief description.

Argument Variables

NK - Number of data points per record.

L1 - Number of sequential searches for base and signal points allowed before cycle is aborted.

L2 - Number of base points required for projected signal point search.

IREC - Record number being processed.

TOL1 - Sets tolerance on base limits for base point selection.

TOL2 - Sets tolerance on signal limits for signal point selection.

TOL3 - Number of signal and base points required for a normal cycle.

NCI - Number of class intervals. See subroutine HIST.

IBTYP - Type base calculation. See subroutine HIST.

Common Block Variables

P - Work array containing record of data points being processed.

AUX - Auxillary array containing the next record to be processed.

INDO - Array containing position in the data array from which the signal points came.

JOVF - Number of overflows in HIST. Variable is initialized in EXT.

JUNF - Number of underflows in HIST. Variable is initialized in EXT.

BASE - Array containing both groups of base points.

SIG - Array containing signal points .

I BLOC - Array containing position in the data array P from which each group of base points came.

L, M - When EXT calls subroutine LIMIT, it gives it the initial position L and the final position M LIMIT is to scan in the data array.

XL90 - This is the result of calling LIMIT and is the criteria for selecting base points.

XL10 - Also the result of LIMIT and is the criteria for selecting signal points.

Labeled Common Variables

IPRINT - If IPRINT is other than 0, EXT will print the record if any irregularities occur while the record is being processed. If IPRINT = 0 it will avoid printing.

IRTN - Place keeper for subroutine EXT. The subroutine may be in any part of its cycle when it completes a record since data is continuous from record to record. IRTN is set to a number corresponding to the exit point in the routine when it returns to the MAIN routine for a new record. When subroutine EXT is recalled, a computed GO TO statement keyed to IRTN returns control to the phase EXT was previously in.

IE - Array passed to subroutine print which contains the errors accumulated for each record.

IBCNT - Array containing the number of base points in each group.

ISCT - Number of signal points.

Important Internal Variables

IRUN - IRUN = 1 indicates routine in start up cycle. Converse for IRUN = 0.

INDX - The value of this variable indicates the position (1-200) in the record being processed.

IDINX - The number of positions subroutine FILL must place zeros due to irregularities which cause data points to be skipped.

NP - Count of unsuccessful passes through signal and base search cycle.

IBC - Indicates which group of base points are being searched for.

LC - Indicates loop condition. LC = 0 means routine is the base point search phase; LC = 1 indicates signal point search.

INP - Counts the times the error recycle phase is entered.

SUBROUTINES HIST (BA, IDUM, NCI, IBTYP, IRUN)

This routine is called by EXT to compute the amplitude of the chopped wave and constructs a histogram with the results. The routine is designed to use numbers between 0 - 1000 but can be easily modified to handle a larger range. The histogram is stored in a common array to be used by the statistical analysis subroutine STAT.

Argument Variables

BA - Average of the base points.

NCI - Number of class intervals for histogram.

IBTYP - Determines the method to be used to calculate the amplitude. If IBTYP = 0, amplitudes are calculated by taking the difference between the signal points and the average of the base points in both groups. If IBTYP = 1, calculation will be the difference between the first signal point and the last base point of group 1, and the difference between the last signal point and the first base point of group 2. If IBTYP = 3, calculation is performed only on the last base point of group 1 and the first signal point.

IRUN - If equal to 1, indicates that EXT is in its first cycle.

Common Block Variables

NCNT - Array containing frequency for each class interval.

JOVF - Number of overflows or excessively large numbers resulting from classifications of amplitudes.

JUNF - Number of underflows or very small numbers resulting from classification of amplitudes.

BASE - Array containing two groups of base points.

SIG - Array containing signal points.

```
FORTRAN IV G LEVEL 1, MOD 4            EXT                DATE = 70122          02/35/57

0001                  SUBROUTINE EXT(NK,L1,L2,IREC,TOL1,TOL2,TOL3,NCI,IBTYP)
0002                  COMMON P(1000),AUX(1000),INDO(10),NCNT(300),JOVF,JUNF,
                     *BASE(2,10),SIG(10),IBLOC(2,10),L,M,XL90,XL10
0003                  COMMON/AAA/IPRINT
0004                  COMMON/RLC/IRTN
0005                  COMMON/PR/IE(7,300)
0006                  COMMON/CBA/IBCNT(2),ISCT
0007                 .IREC=IREC+1
0008                  IST=IST + 1
0009                  INP=0
0010                  INO=1
0011                  INDX=1
0012                  GO TO (150,11,12,150,776),IRTN
0013           150    IRUN=1
0014                  IRP=0
0015                  IST=0
0016                  L=1
0017                  M=20
0018                  IF(IRTN.EQ.4)  GO TO 209
0019                  DO 1050 I=1,300
0020          1050    NCNT(I)=0
0021                  JOVF=0
0022                  JUNF=0
0023                  NDATA=0
0024           209    CALL LIMIT(TOL1,TOL2,NK)
                     C      SEARCH FOR FIRST BASE POINT- HOLD VALUE OF INDX
0025           776    DO 1 I=L,M
0026                  IF(I.GT.NK)  GO TO 2000
0027                  IF(P(I)-XL90) 1,3,3
0028           3      INDX=I
0029                  GO TO 4
0030           1      CONTINUE
0031                  IE(1,IREC)=IE(1,IREC)+1
0032                  IRTN=4
                     C      RETURN TO DRIVER FOR NEXT RECORD
0033                  RETURN
0034           2000   L=1
0035                  M=M-NK
0036                  IRTN=5
0037                  RETURN
0038              4   M=INDX+15
0039                  L=INDX
0040                  CALL LIMIT(TOL1,TOL2,NK)
0041                  IDINX=IABS(INDX-INO)
0042                  CALL FILL(IDINX)
0043                  INO=INDX
0044                  IBC=1
0045                  IBCNT(1)=0
0046                  IBCNT(2)=0
0047                  ISCT=0
0048           10     LC=0
0049                  NP=0
                     C      BASE POINT SEARCH
0050           11     IF(P(INDX)-XL90)  12,13,13
0051           13     IBCNT(IBC)=IBCNT(IBC)  +1
0052                  I=IBCNT(IBC)
```

```
FORTRAN IV G LEVEL 1, MOD 4            EXT              DATE = 70122          02/35/57

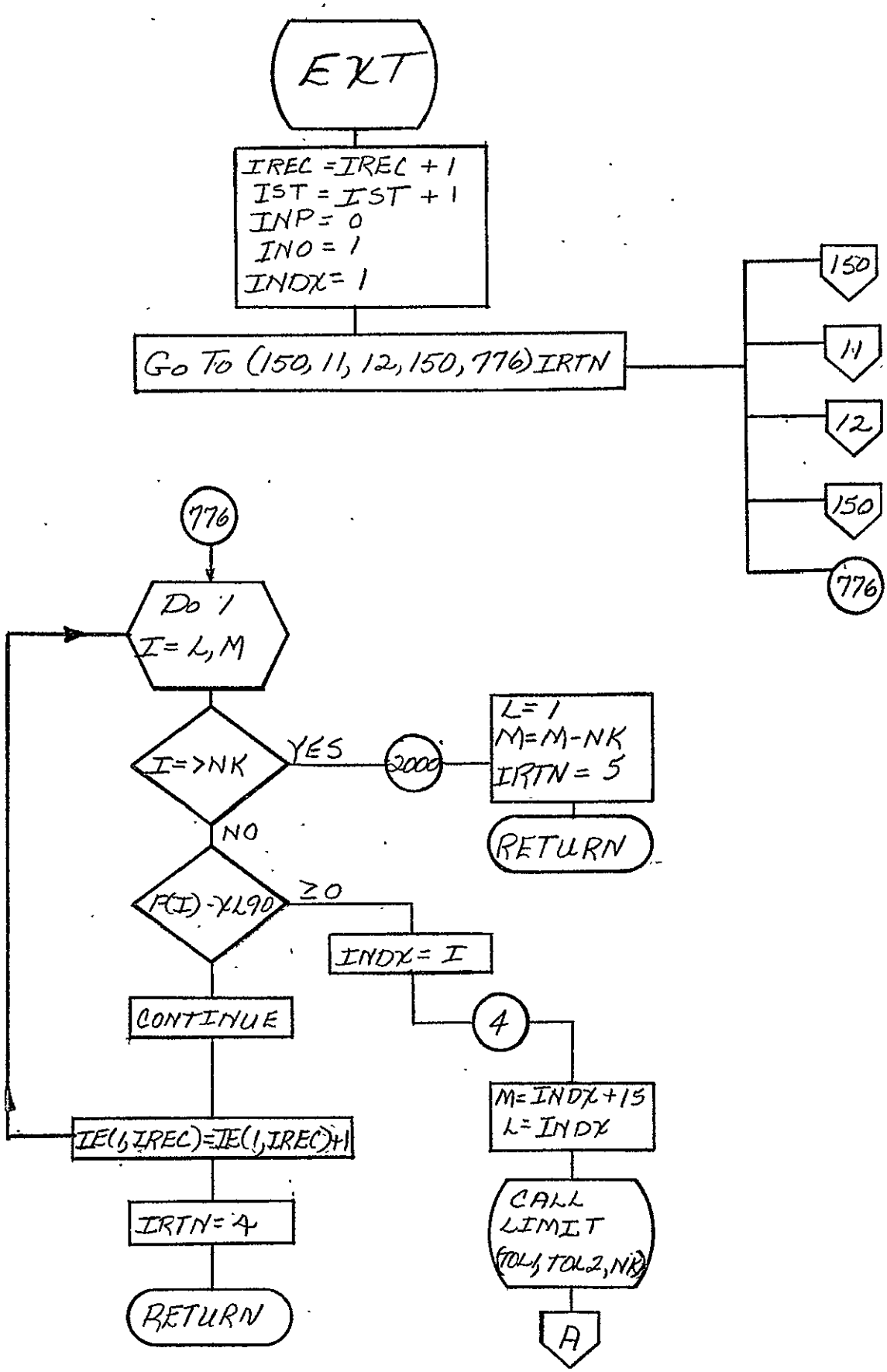
              C     STORE BASE POINT
0053                BASE(IBC,I)=P(INDX)
0054                IBLOC(IBC,I)=INDX
0055                IF(IBCNT(IBC)-10) 14,401,401
0056           401  IRTN=4
0057                IE(2,IREC)=IE(2,IREC)+1
0058                IDINX=NK-INDX
0059                CALL FILL(IDINX)
0060                RETURN
0061           14   INDX=INDX + 1
0062                INO=INDX
0063                IF(INDX-NK) 11,11,158
0064           158  IRTN=2
              C     NORMAL RETURN TO DRIVER FOR NEXT RECORD
0065                RETURN
              C     SIGNAL POINT SEARCH
0066           12   IF(P(INDX)-XL10) 20,20,15
0067           15   IF(LC) 16,16,21
0068           16   NP=NP+1
              C     CHECK NUMBER OF UNSUCESSFUL PASSES
0069                IF(NP-L1) 1944, 1944, 208
0070           1944 IF(ISCT.EQ.0) GO TO 14
0071                GO TO 70
0072           21   IF(P(INDX)-XL90) 22,30,30
0073             30 IF(IRUN) 10,10,31
              C     SET BASE COUNT INDX CONDITION
0074           31   IF(IBC-1)34,33,34
0075           33   IBC=2
0076                GO TO 10
0077           34   IBC=1
0078           20   IF(IRUN) 23,23,24
              C     CHECK LOOP CONDITION
0079           23   IF(LC) 40,40,29
0080           24   IF(IBC-1) 25,25,40
0081           25   LC=1
0082                NP=0
0083           29   ISCT=ISCT + 1
              C     STORE SIGNAL POINT
0084                SIG(ISCT)=P(INDX)
0085                INDO(ISCT)=INDX
0086                IF(ISCT-10) 26,501,501
0087           501  IRTN=4
0088                IE(3,IREC)=IE(3,IREC)+1
0089                IDINX=NK-INDX
0090                CALL FILL(IDINX)
0091                RETURN
0092           26   INDX=INDX+1
0093                INO=INDX
0094                IF(INDX-NK) 12,12,162
0095           162  IRTN=3
              C     NORMAL RETURN TO DRIVER FOR NEXT RECORD
0096                RETURN
0097           22   NP=NP+1
0098                IF(NP-L1) 26,26,208
              C     CHECK NUMBER OF BASE AND SIGNAL POINTS
0099           40   IF(IBCNT(1).LT.TOL3) GO TO 222
```

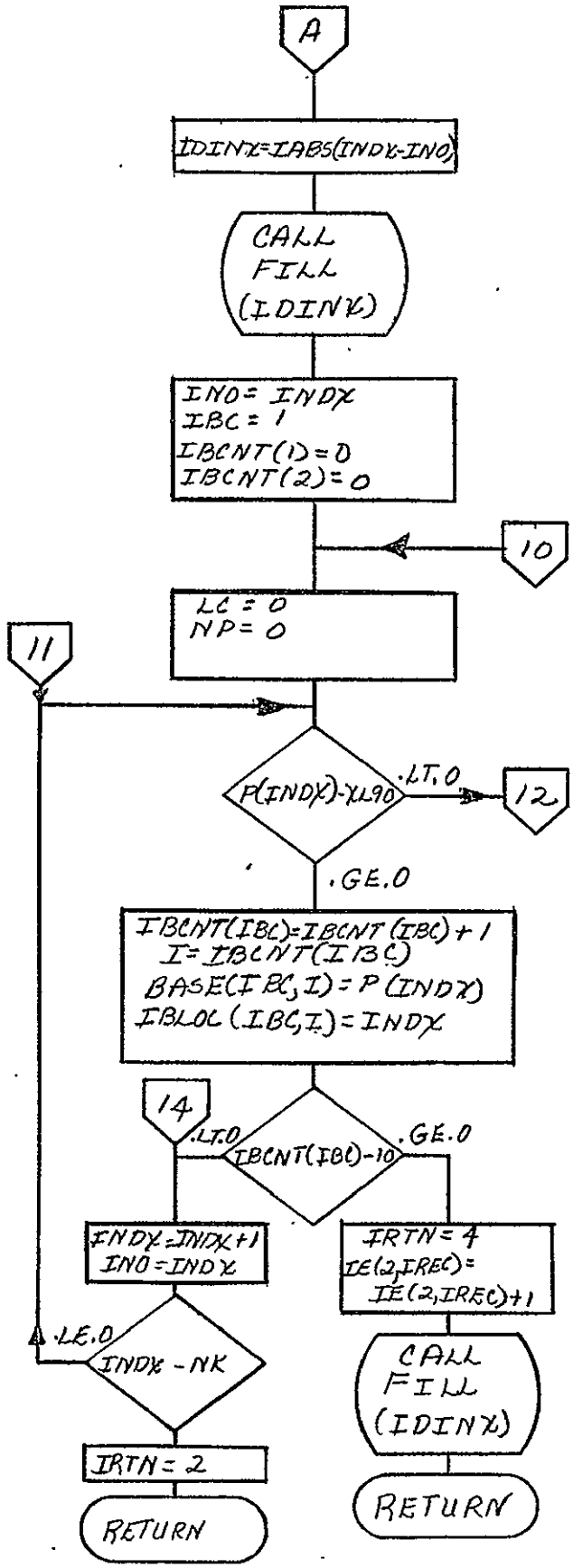
```
FORTRAN IV G LEVEL 1, MOD 4            EXT                DATE = 70122          02/35/57

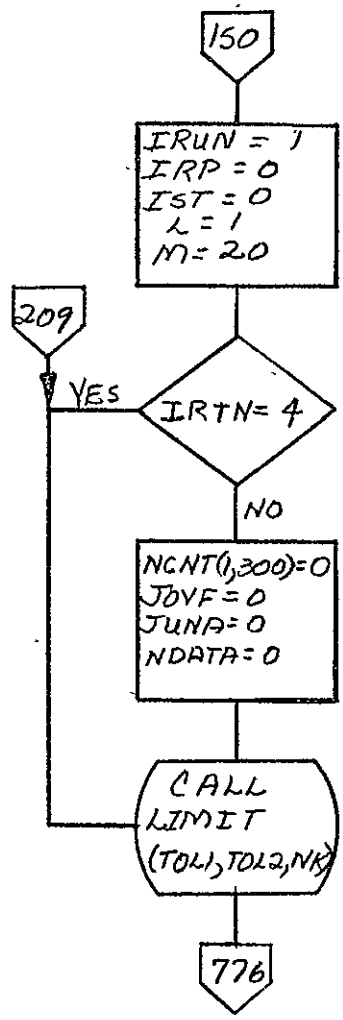
0100                IF(IBCNT(2).LT.TOL3) GO TO 222
0101                IF(ISCT.LT.TOL3) GO TO 224
0102                GO TO 100
0103              222 IE(4,IREC)=IE(4,IREC)+1
0104                GO TO 555
0105              224 IE(5,IREC)=IE(5,IREC)+1
0106                GO TO 555
                  C     ENTER ERROR RECYCLE
0107              208 IE(6,IREC)=IE(6,IREC)+1
0108              555 L=INDX
0109                  M=INDX+20
0110                  IRUN=1
0111                  IF(IPRINT.EQ.0) GO TO 742
0112                  PRINT 666,IREC,INDX
0113              666 FORMAT(1X,'IREC=',I10,10X,'INDX=',I10)
0114                  IF(IREC.EQ.IRP) GO TO 742
0115                  PRINT 333,(P(I),I=1,NK)
0116              333 FORMAT(1X,/,(1X,10F10.3))
0117                  IRP=IREC
0118              742 CONTINUE
0119                  INP=INP+1
0120                  IF(INP-5)209,921,921
0121              921 IE(7,IREC)=IE(7,IREC)+1
0122                  IRTN  = 4
0123                  IDINX=NK-INDX
0124                  CALL FILL(IDINX)
0125                  RETURN
                  C     POINT SEARCH CYCLE COMPLETE
0126              100 SUM1=0.0
0127                  SUM2=0.0
0128                  I=IBCNT(1)
                  C     PREPROSSING FOR HISTOGRAM FOLLOWS
0129                  DO 101 J=1,I
0130              101 SUM1=SUM1+ BASE(1,J)
0131                  I=IBCNT(2)
0132                  DO 102 J=1,I
0133              102 SUM2=SUM2 +BASE(2,J)
0134                  BA=(SUM1+SUM2)/(IBCNT(1) +IBCNT(2))
0135                  CALL AMPX(IRUN)
0136                  CALL HIST(BA,NK,NCI,IBTYP,IRUN)
0137               43 IF(IBC-1) 44,45,44
0138               45 IBC=2
0139                  GO TO 46
0140               44 IBC=1
0141               46 ISCT=0
0142                  IBCNT(IBC)=0
0143                  LC=1
0144                  NP=0
0145                  IRUN=0
0146                  INDX2=INDX+9
0147                  L=INDX
0148                  M=INDX2
0149                  CALL LIMIT(TOL1,TOL2,NK)
0150                  GO TO 12
                  C     LOOK INTO NEXT RECORD FOR SIGNAL POINT
0151               70 IF(IBCNT(IBC)-L2) 14,72,72
```

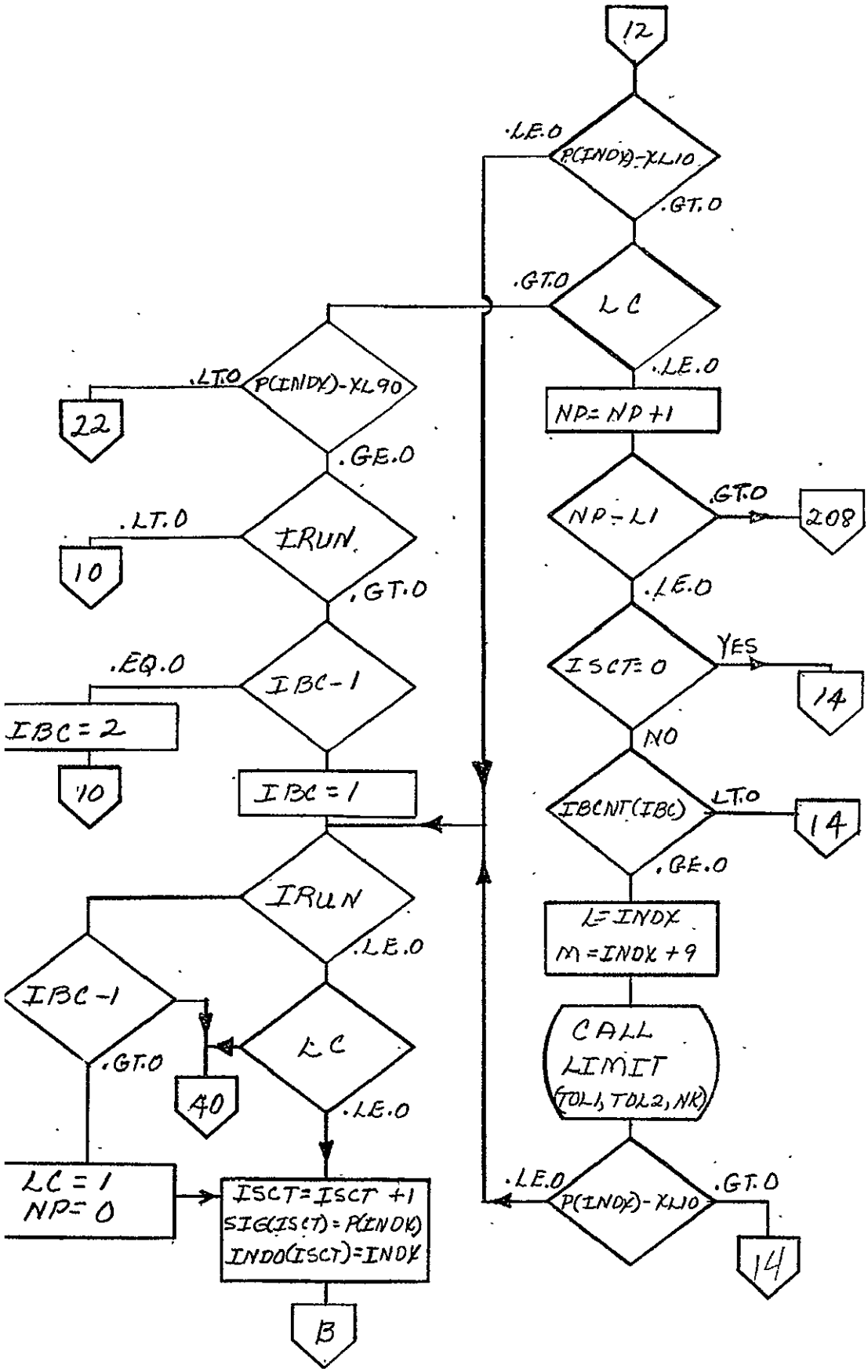
```
FORTRAN IV G LEVEL 1, MOD 4            EXT              DATE = 70122          02/35/57

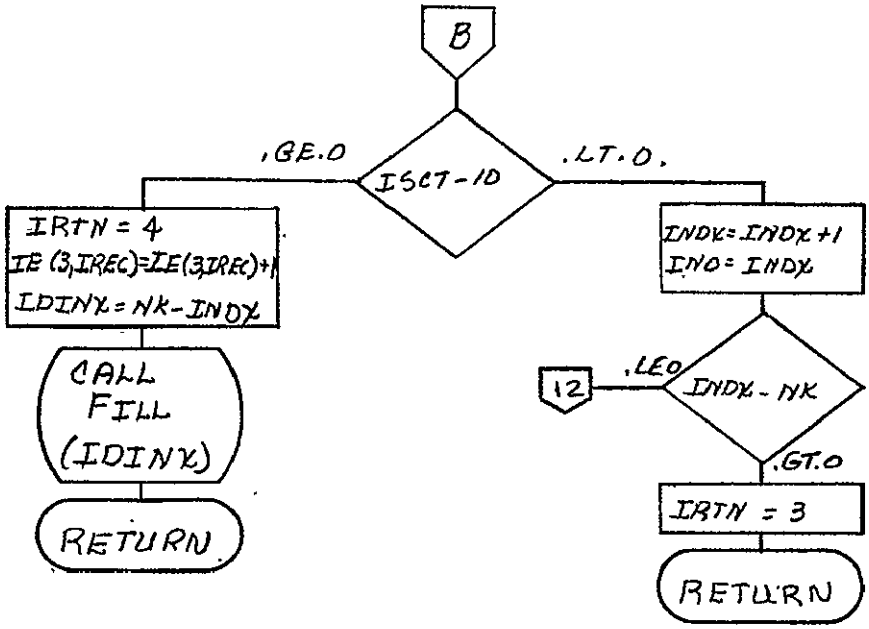
 0152          72    L=INDX
 0153                M=INDX + 9
 0154                CALL LIMIT(TOL1,TOL2,NK)
 0155                IF(P(INDX)-XL10) 20,20,14
 0156                END
```

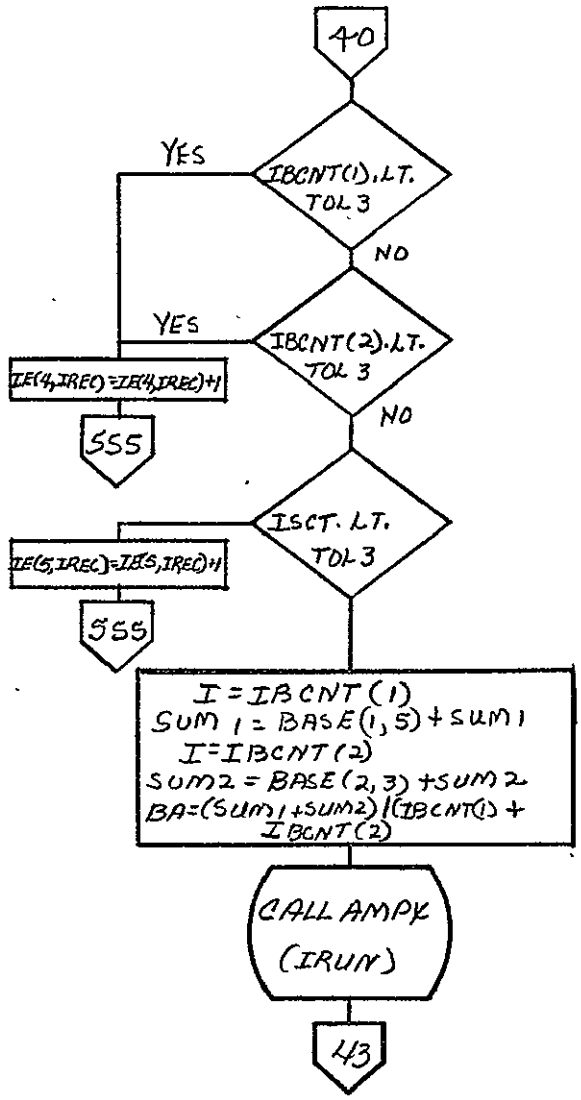

EXT
IREC = IREC + 1
IST = IST + 1
INP = 0
INO = 1
INDX = 1
GO TO (150, 11, 12, 150, 776) IRTN
150
11
12
150
776
776
DO 1
I = L, M
I => NK
YES
2000
L = 1
M = M - NK
IRTN = 5
RETURN
NO
F(I) - XL90
≥ 0
INDX = I
4
CONTINUE
M = INDX + 15
L = INDX
IE(1, IREC) = IE(1, IREC) + 1
CALL LIMIT (TOL1, TOL2, NK)
IRTN = 4
RETURN
A

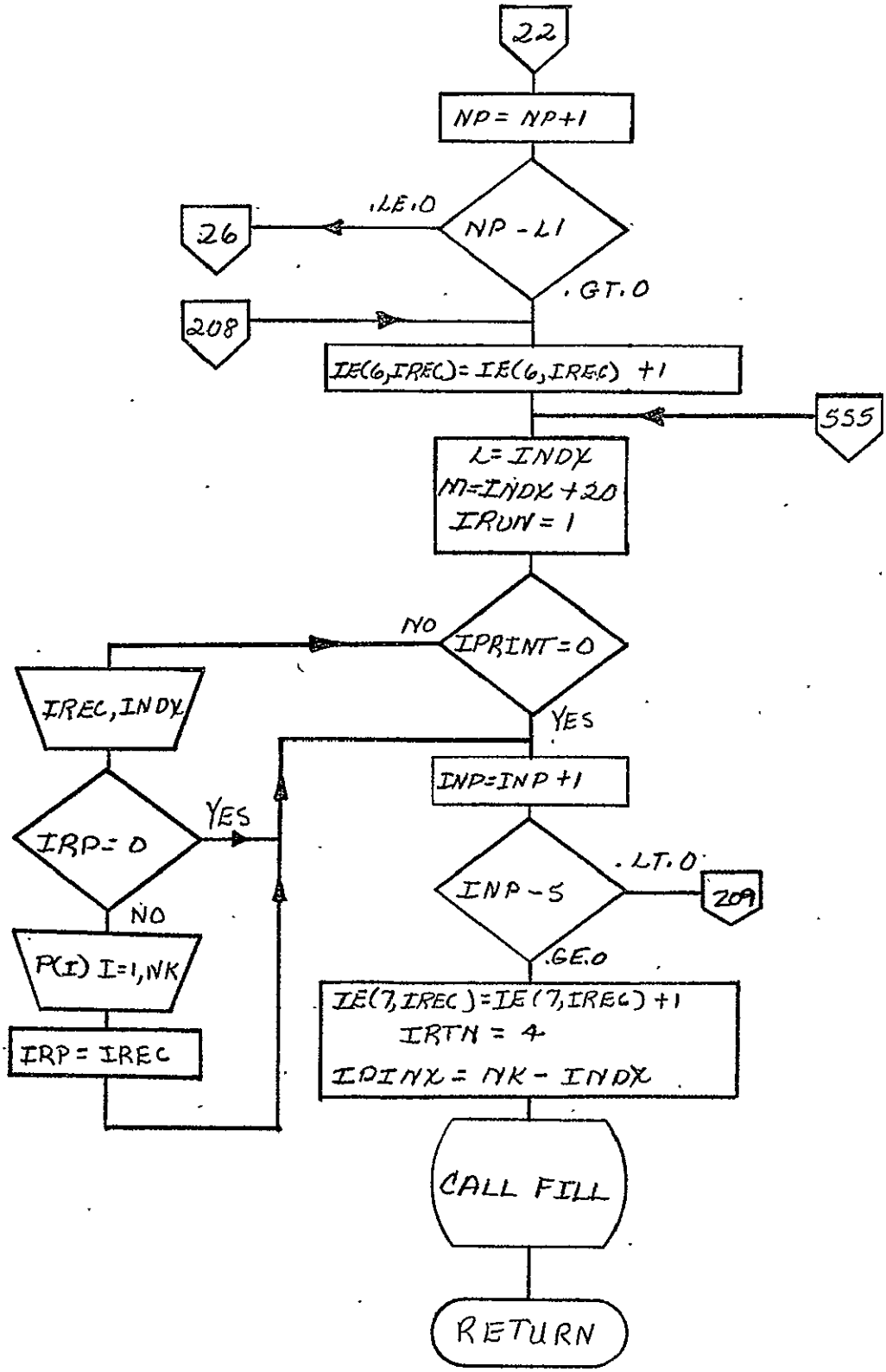

A
IDINX=IABS(INDX-INO)
CALL FILL (IDINX)
INO=INDX
IBC=1
IBCNT(1)=0
IBCNT(2)=0
10
LC=0
NP=0
11
P(INDX)-XL90
.LT.0
12
.GE.0
IBCNT(IBC)=IBCNT(IBC)+1
I=IBCNT(IBC)
BASE(IBC,I)=P(INDX)
IBLOC(IBC,I)=INDX
14
.LT.0
IBCNT(IBC)-10
.GE.0
INDX=INDX+1
INO=INDX
.LE.0
INDX-NK
IRTN=2
RETURN
IRTN=4
IE(2,IREC)=IE(2,IREC)+1
CALL FILL (IDINX)
RETURN

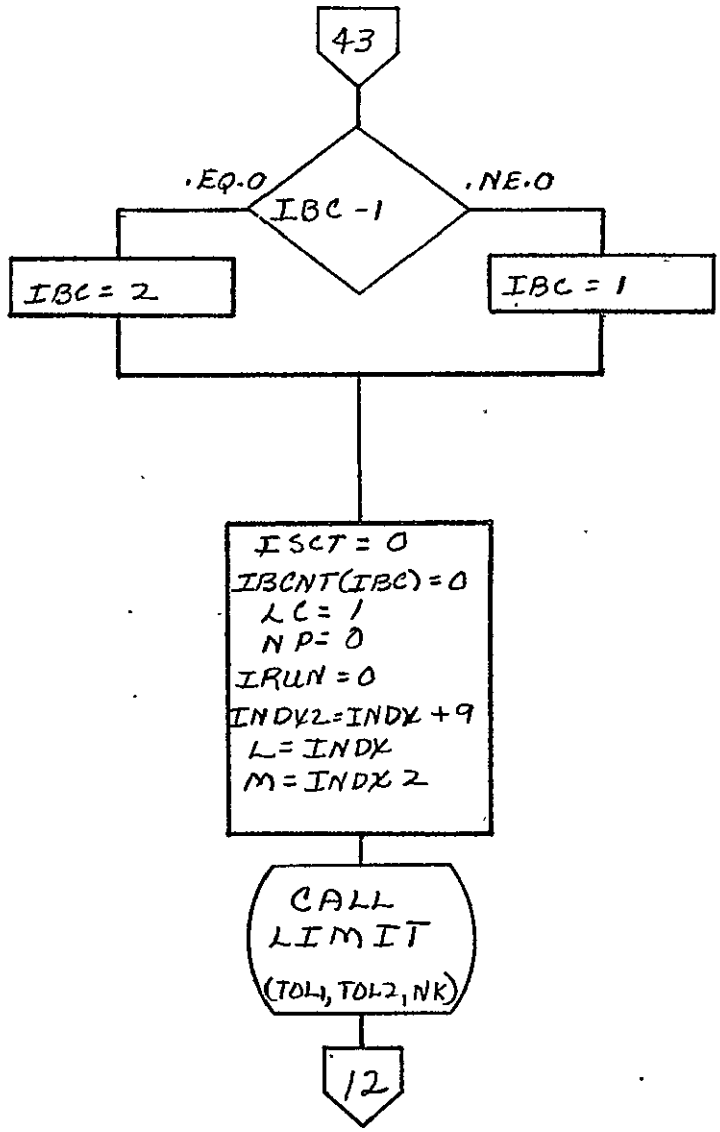

150
IRUN=1
IRP=0
IST=0
L=1
M=20
209
YES
IRTN=4
NO
NCNT(1,300)=0
JOVF=0
JUNA=0
NDATA=0
CALL LIMIT (TOL1,TOL2,NK)
776


12
.LE.0
P(INDX)-XL10
.GT.0
.GT.0
LC
.LE.0
.LT.0
P(INDX)-XL90
22
.GE.0
NP=NP+1
.LT.0
IRUN
10
.GT.0
NP-L1
.GT.0
208
.LE.0
.EQ.0
IBC-1
ISCT=0
YES
14
IBC=2
10
NO
IBC=1
IBCNT(IBC)
.LT.0
14
.GE.0
IRUN
L=INDX
M=INDX+9
.LE.0
IBC-1
.GT.0
LC
CALL
LIMIT
(TOL1, TOL2, NK)
40
.LE.0
LC=1
NP=0
ISCT=ISCT+1
SIG(ISCT)=P(INDX)
INDO(ISCT)=INDX
.LE.0
P(INDX)-XL10
.GT.0
14
B


B
ISCT-10
.GE.0
.LT.0.
IRTN = 4
IE(3,IREC)=IE(3,IREC)+1
IDINX = NK-INDX
CALL FILL (IDINX)
RETURN
INDX=INDX+1
INO = INDX
12
.LE.0
INDX - NK
.GT.0
IRTN = 3
RETURN


40
YES
IBCNT(1).LT. TOL 3
NO
YES
IBCNT(2).LT. TOL 3
NO
IE(4,IREC)=IE(4,IREC)+1
555
ISCT. LT. TOL 3
IE(5,IREC)=IE(5,IREC)+1
555
I = IBCNT(1)
SUM1 = BASE(1,5)+SUM1
I=IBCNT(2)
SUM2 = BASE(2,3)+SUM2
BA=(SUM1+SUM2)/(IBCNT(1)+IBCNT(2)
CALL AMPX (IRUN)
43


22
NP = NP+1
.LE.0
NP - L1
26
.GT.0
208
IE(6,IREC)= IE(6,IREC) +1
555
L= INDX
M=INDX +20
IRUN = 1
NO
IPRINT = 0
IREC, INDX
YES
INP=INP +1
IRP= 0
YES
.LT.0
INP - 5
209
NO
.GE.0
P(I) I=1,NK
IE(7,IREC)=IE (7,IREC) +1
IRTN = 4
IRP = IREC
IDINX = NK - INDX
CALL FILL
RETURN


43
IBC -1
.EQ.0
.NE.0
IBC = 2
IBC = 1
ISCT = 0
IBCNT(IBC) = 0
LC = 1
NP = 0
IRUN = 0
INDX2 = INDX + 9
L = INDX
M = INDX 2
CALL
LIMIT
(TOL1, TOL2, NK)
12

```
FORTRAN IV G LEVEL 1, MOD 4            HIST              DATE = 70122          02/35/57

0001                  SUBROUTINE HIST(BA,IDUM,NCI,IBTYP,IRUN)
0002                  COMMON P(1000),AUX(1000),INDO(10),NCNT(300),JOVF,JUNF,
                     *BASE(2,10),SIG(10),IBLOC(2,10),KK,MM,XL90,XL10
0003                  DIMENSION Y(40), X(40), LOC(40), A(15), V(15), B(200)
0004                  COMMON/CBA/IBCNT(2),N
                C     COMBINE BASE 1 AND BASE 2
0005                  IF(IBTYP) 100,100,200
0006                200 IF(IRUN.EQ.1) MMM=1
0007                  KKK=1
0008                  LLL=1
0009                  IF(MMM.EQ.1) LLL=2
0010                  IF(MMM.EQ.0) KKK=2
0011                100 IJ=N
0012                  DO 1 I=1,N
0013                  IF(IBTYP.EQ.0) GO TO 202
0014                400 IF(I-1) 402,401,402
0015                401 NTEMP=IBCNT(KKK)
0016                  BA=BASE(KKK,NTEMP)
0017                  GO TO 202
0018                402 IF(I-IJ) 1,403,1
0019                403 BA=BASE(LLL,1)
0020                202 J=BA-SIG(I)
0021                  J=(NCI*J)/1000    +1
0022                  IF(J-1) 2,3,3
0023                2   JUNF=JUNF+1
0024                  GO TO 1
0025                3   IF(J-300) 5,5,4
0026                4   JOVF=JOVF+1
0027                  GO TO 1
0028                5   NDATA=NDATA+1
0029                  NCNT(J)=NCNT(J)+1
0030                  IF(IBTYP.EQ.3) GO TO 50
0031                  1 CONTINUE
0032                  IF(MMM.EQ.0) MMM=1
0033                  IF(MMM.EQ.1) MMM=0
0034                 50 RETURN
0035                  END
```

SUBROUTINE STAT (NTAPE, NCH, NFILE, NCI, IFLAG)

STAT performs the statistical analysis by using the intensity histogram constructed by HIST. A log-amplitude histogram is generated from the intensity histogram to perform log-normal tests. The mean, standard deviation, skewness and kurtosis are calculated for both the intensity and log-amplitude data. In addition, the cumulative probability is calculated and a chi-square test made on both sets of data.

Argument Variables

NTAPE - Tape number.

NCH - Channel number.

NFILE - File number.

NCI - Number of class intervals.

IFLAG - Indicates type statistical calculations.

Common Block Variables

NCNT - Array contains histogram.

JOF - Number of overflows.

JUF - Number of underflows.

SUBROUTINE LIMIT (TOL1, TOL2, NK)

This routine is called by subroutine EXT to calculate the criteria for determining if a data point is a base point or a signal point or neither. LIMIT has the capability of looking into the next record if it is called near the end of the record being processed.

Argument Variables

TOL1 - Experimentally determined constant.

TOL2 - Experimentally determined constant.

NK - Number of data points in record.

```
FORTRAN IV G LEVEL 1, MOD 4            STAT              DATE = 70122          02/35/57

0001                  SUBROUTINE STAT(NTAPE,NCH,NFILE,NCI,IFLAG)
                C      IFLAG = I   NO CHI SQUARE TEST
                C              2   CHI SQUARE TEST ON NORMAL DISTRIBUTION
                C              3  CHI SQUARE TEST ON LOG NORMLL DISTRIBUTION
                C              4  CHI SQUARE TEST ON BOTH
0002                  COMMON P(1000),AUX(1000),INDO(10),NCNT1(300),JOF,JUF,BASE(2,10),
                     *PIG(10),IBLQC(2,10),KK,NM,XL90,XL10
0003                  COMMON/UUU/NPNCH
0004                  DIMENSION Y(300),XLN(300),Q(4),R(4)
0005                  DIMENSION NCNT(300)
0006                  DO 60 I=1,300
0007                60 NCNT(I)=NCNT1(I)
0008                  C=1000/NCI
                C       FIND THE HIGHEST  AND LOWEST CLASS INTERVAL
0009                  DO 1 I=1,NCI
0010                  IF(NCNT(I)) 1,1,2
0011                2 ILO=I
0012                  GO TO 3
0013                1 CONTINUE
0014                3 DO 4 I= ILO,NCI
0015                  IF(NCNT(I))4,4,5
0016                5 IHI=I
0017                4 CONTINUE
                C       FIND NUMBER OF POINTS AND FLOAT NCNT
0018                  N=0
0019                  DO 6 I=ILO,IHI
0020                  Y(I)=NCNT(I)
0021                6 N=N+NCNT(I)
                C      PRINT HEADINGS
0022                  WRITE(6,101) NTAPE,NCH,NFILE
0023                  WRITE(6,102)
                C      DEFALT DUE TO TOO FEW CLASS INTERVALS
0024                  NN= IHI-ILO
0025                  IF(NN-10) 50,50,51
0026               50 WRITE(6,110)
0027                  RETURN
                C      FIND AVERAGE AMPLITUDE
0028               51 XN=N
0029                  AVE=0.00
0030                  DO 8 I=ILO,IHI
0031                  XI=I
0032                8 AVE=AVE+Y(I)*(XI-0.5)*C
0033                  AVE=AVE/XN
                C      COMPUTE CUMULATIVE PROBABILITIEC AND LOG AMPLITUDES
0034                  SUM=0.00
0035                  DO 10 I=ILO,IHI
0036                  XI=I
0037                  XI=(XI-0.5)*C
0038                  XLN(I)=0.5*ALOG(XI/AVE)
0039                  SUM=SUM+Y(I)
0040                  CP=SUM/XN
0041               10 WRITE(6,103)  XI,XLN(I),Y(I),CP
0042                  WRITE(6,111) JOF, JUF
                C      COMPUTE MOMENTS ABOUT THE MEAN
0043                  XLA=0.00
0044                  DO 20 I=ILO,IHI
```

```
FORTRAN IV G LEVEL 1, MOD 4            STAT              DATE = 70122          02/35/57

0045            20 XLA=XLA+Y(I)*XLN(I)
0046               XLA=XLA/XN
0047               DO 21 I=2,4
0048               Q(I)=0.00
0049            21 R(I)=0.00
0050               DO 22 I=ILO,IHI
0051               XI=I
0052               DO 22 J=2,4
0053               Q(J)=Q(J)+Y(I)*((XI-0.5)*C-AVE)**J
0054            22 R(J)=R(J)+Y(I)*(XLN(I)-XLA)**J
0055               DO 23 J=2,4
0056               Q(J)=Q(J)/XN
0057            23 R(J)=R(J)/XN
0058               NT=IHI-ILO+1
0059               WRITE(6,104) NT
0060               SIG=SQRT(Q(2))
0061               SIGL=SQRT(R(2))
0062               SKEW=0.5*Q(3)/(SIG**3)
0063               SKL = 0.5*R(3)/(SIGL**3)
0064               XKUR=((Q(4)/(Q(2)**2))-3.0)/2.0
0065               XKURL = ((R(4)/(R(2)**2))-3.0)/2.0
                C       PRINT MOMENTS
0066               WRITE(6,105) AVE,SIG,SKEW,XKUR,XLA,SIGL,SKL,XKURL,N
0067               IF(NPNCH .EQ. 0) GO TO 810
0068               PUNCH 800,NTAPE,NCH,NFILE,AVE,SIG,SIGL,XLA
0069           800 FORMAT(A4,I2,I4,4E14.4)
0070           810 CONTINUE
0071               GO TO (31,32,42,32) ,IFLAG
0072            31 RETURN
                C       NO CHI SQUARE TEST REQUESTED
                C
                C       CHI SQUARE TEST
0073            32 CALL CHI(CSQ,Y,ILO,IHI,C,NUSE,AVE,XLA,SIG ,XN,0,SIG)
0074               XN=CSQ
0075               WRITE(6,106) CSQ,NUSE
                C       PRINT CHI SQUARE     NORMAL
0076               GO TO (31,31,42,42)     ,IFLAG
0077            42 CALL CHI(CSQ,Y,ILO,IHI,C,NUSE,AVE,XLA,SIG ,XN,1,SIGL)
0078               AXLN=CSQ
0079               WRITE(6,106) CSQ,NUSE
                C       PRINT CHI SQUARE     LOG NORMAL
0080               PUNCH 999,NTAPE,NCH,NFILE,SIGL,SKEW,SKL,XN,AXLN
0081           999 FORMAT(A4,I1,I2,5E13.6)
0082               RETURN
0083           101 FORMAT('1',5X,'TAPE NUMBER  ',A4,5X,'TRACK',I3,5X,'FILE',I3)
0084           102 FORMAT('0',16X,'AMPLITUDE',10X,'LOG AMPLITUDE',12X,'COUNT',
                  1  8X,'CUMULATIVE PROBABILITY'/)
0085           103 FORMAT(7X,4E21.6)
0086           104 FORMAT('0',10X,'NUMBER OF CLASS INTERVALS =',I6)
0087           105 FORMAT('0',17X,'AVERAGE',9X,'STANDARD DEVIATION',8X,'SKEWNESS',
                  *13X,'KURTOSIS'/7X,4E21.6/7X,4E21.6/'0',10X,'NUMBER OF DATA POINTS'
                  *,I10)
0088           106 FORMAT('0',10X,'CHI SQUARE=',E14.6/11X'NUMBER OF CLASS INTERVALS
                  1USED =',I5)
0089           110 FORMAT('0',5X,'TOO FEW CLASS INTERVALS  '/
                  1 '0',5X,'  EXECUTION OF STATISTICS CALCULATION SUSPENDED')
```

```
FORTRAN IV G LEVEL 1, MOD 4            STAT              DATE = 70122          02/35/57

 0090          111  FORMAT('0',5X,'NUMBER OF OVERFLOWS',I6/
                    *6X,'NUMBER OF UNDERFLOWS',I5)
 0091               END
```

Common Block Variables

KK - Gives the position (value of INDX) in the record at the time LIMIT is called.

MM - This variable is the sum of KK and the number of points LIMIT is to scan.

XL90 - The resulting criteria for base point selection.

XL10 - The resulting criteria for signal point selection.

SUBROUTINE AMPX (IRUN)

This routine takes the signal and base points extracted by EXT and computes the amplitude of the square wave for the spectral analysis. The amplitudes are calculated by taking the difference between the last base point in the first group and the first signal point, and the difference between the last signal point and the first base point in the second group. This produces two signal points per group. The points are stored in array AMP for use by the spectral analysis routines.

Argument Variables

IRUN - Indicates if EXT in startup cycle.

Common Block Variables

BASE - Array containing both groups of base points.

SIG - Array containing signal points.

Labeled Common Variables

IBCNT - Array containing number of base points in each group.

N - Number of signal points

SUBROUTINE CHI (CSQ, Y1, ILO, IHI, C, NUSE, AVE, XLA, SD, XN, NTYP, SX)

This routine is called by the statistics subroutine to perform the chi-square test for normal and log-normal distributions.

```
FORTRAN IV G LEVEL 1, MOD 4            LIMIT              DATE = 70122          02/35/57

0001                SUBROUTINE LIMIT(TOL1,TOL2,NK)
0002                COMMON P(1000),AUX(1000),INDO(10),NCNT(300),JOVF,JUNF,
                   *BASE(2,10),SIG(10),IBLOC(2,10),KK,MM,XL90,XL10
0003                IK=KK
0004                IM=MM
0005                ID=0
0006                IF(IK-NK) 502,501,666
0007           666  PRINT 3000
0008           3000 FORMAT('  IK GREATER THAN NK    ')
0009           501  AMAX=P(IK)
0010                AMIN=AMAX
0011                ID=IM-NK
0012                GO TO 381
0013           502  IF(IM-NK) 360,360,361
0014           361  ID=IM-NK
0015                IM=NK
0016           360  AMAX=P(IK)
0017                AMIN=AMAX
0018                DO 350 J=IK,IM
0019                IF(AMAX-P(J)) 301,302,302
0020           301  AMAX=P(J)
0021           302  IF(AMIN-P(J)) 350,303,303
0022           303  AMIN=P(J)
0023           350  CONTINUE
0024                IF(ID) 380,380,381
0025           381  AMAXP=AUX(1)
0026                AMINP=AMAXP
0027                DO 650 J=1,ID
0028                IF(AMAXP-AUX(J)) 601,602,602
0029           601  AMAXP=AUX(J)
0030           602  IF(AMINP-AUX(J)) 650,603,603
0031           603  AMINP=AUX(J)
0032           650  CONTINUE
0033                IF(AMAX-AMAXP)700,701,701
0034           700  AMAX=AMAXP
0035           701  IF(AMIN-AMINP)380,380,703
0036           703  AMIN=AMINP
0037           380  A=AMAX-AMIN
0038                XL90=AMAX-TOL1*A
0039                XL10=AMIN+ TOL2*A
0040                RETURN
0041                END
```

```
FORTRAN IV G LEVEL 1, MOD 4            AMPX              DATE = 70122          02/35/57

0001                  SUBROUTINE AMPX(IRUN)
0002                  COMMON P(1000),AUX(1000),INDO(10),NCNT(300),JOVF,JUNF,
                     *BASE(2,10),SIG(10),IBLOC(2,10),KK,MM,XL90,XL10
0003                  COMMON/CBA/IBCNT(2),N
0004                  COMMON/AP/AMP(10000),NDATA
0005                  IF(IRUN.EQ.1) MMM=1
0006                  KKK=1
0007                  LLL=1
0008                  IF(MMM.EQ.1) LLL=2
0009                  IF(MMM.EQ.0) KKK=2
0010                  IJ=N
0011                  DO 1 I=1,N
0012            400   IF(I-1) 402,401,402
0013            401   NTEMP=IBCNT(KKK)
0014                  BA=BASE(KKK,NTEMP)
0015                  GO TO 502
0016            402   IF(I-IJ) 1,403,1
0017            403   BA=BASE(LLL,1)
0018            502   NDATA=NDATA+1
0019                  AMP(NDATA)=BA*SIG(I)
0020            1     CONTINUE
0021                  IF(MMM.EQ.0) MMM=1
0022                  IF(MMM.EQ.1) MMM=0
0023                  RETURN
0024                  END
```

```
FORTRAN IV G LEVEL 1, MOD 4            CHI                DATE = 70122          02/35/57

0001                  SUBROUTINE CHI(CSQ,Y1,ILO,IHI,C,NUSE,AVE,XLA,SD,XN,NTYP,SX)
0002                  DIMENSION Y(300,3) , Y1(300)
0003                  DO 1 I=1,300
0004                1 Y(I,1)=0.00
0005                  NUSE=0
0006                  KM=(ILO+IHI)/2
0007                  J=1
0008                  Y(J,2)=AVE-10.0*SD
                        GROOP CLASS INTERVALS ON LOW END
0009                  DO 2 I= ILO,KM
0010                  Y(J,1)=Y1(I) + Y(J,1)
0011                  IF(Y(J,1)-5.0) 2,2,3
0012                3 Y(J,3)=C*I
0013                  NUSE = NUSE + 1
0014                  J = J + 1
0015                  Y(J,2) = C*I
0016                2 CONTINUE
                C       GROOP CLASS INTERVALS ON HIGH SIDE
0017                  I = IHI
0018                  Y(J,3) = AVE + 10.0*SD
0019                6 IF(I-KM) 10,10,4
0020                4 Y(J,1) = Y(J,1) + Y1(I)
0021                  IF(Y(J,1)-5.0) 11,11,5
0022                5 Y(J,2) = C*(I-1)
0023                  NUSE = NUSE + 1
0024                  J = J + 1
0025                  Y(J,3) = C*(I-1)
0026               11 I = I-1
0027                  GO TO 6
                C       COMPUTE THEORITICAL PROBABILITY
0028               10 CSQ = 0.00
0029                  DO 30 I=1,NUSE
0030                  XLL = Y(I,2)
0031                  XUL=Y(I,3)
0032               24 CALL SIMP( PTH,XLL,XUL,21,NTYP,AVE,SX,XLA)
                C       COMPUTE CHI SQUARE
0033                  IF(PTH) 31,31,30
0034               31 WRITE(6,100) I
0035              100 FORMAT('0    ZERO VALUE OF THEORITICAL PROBABILITY IN',I5,' TH
                     1 INTERVAL'/ 6X,'EXECUTION OF CHI SQUARE TEST DISCONTINUED')
0036                  RETURN
0037               30 CSQ=CSQ+((Y(I,1)-XN*PTH)**2)/(XN*PTH)
0038                  RETURN
0039                  END
```

Argument Variables

CSQ - Result of chi-square test.

Y1 - Array containing histogram.

ILO - Lowest class interval in histogram.

IHI - Highest class interval in histogram.

C - Width of class mark.

NUSE - Number of class intervals used by the chi-square routine.

AVE - Mean value of amplitudes.

XLA - Mean value of log-amplitudes.

SD - Standard deviation of amplitudes.

XN - Number of data points.

NTYP - Determines if chi-square test will be run for normal or log-normal test or both.

SX - Log standard deviation.

SUBROUTINE FFT (DT, FMAX)

This routine is called by the main program to coordinate the performance of the spectral analysis. Subroutines FOURT and PLOT are called to perform the Fourier transform and to plot the results. FFT will have PLOT plot directly from the calculated spectral data array or it will have it plot the average of a designated number of points in the array. This feature was incorporated to smooth out random variations.

Argument Variables

DT - Time between data samples.

FMAX - Maximum frequency to be used.

Labeled Common Variables

AMP - Array containing the time domain signal. This array is passed from AMPX.

```
FORTRAN IV G LEVEL 1, MOD 4            FFT                DATE = 70122          02/35/57

0001                SUBROUTINE FFT(N,DT,FMAX)
0002                DIMENSION WORK(2500)
0003                COMMON/AP/AMP(10000),NDATA
0004                COMMON/GO/N1,N3,N4,FM,IDO
0005                COMMON/NPTS8/N8
0006                DIMENSION NN(2)
0007                PRINT 555,NDATA
0008               .IF(NDATA-8192) 25,56,56
0009          25    NDIFF=8192-NDATA
0010                CALL FILL(NDIFF)
0011          56    PRINT 81,N8
0012          81    FORMAT(10X,'NUMBER OF ZEROS ADDED  =  ',I20//)
0013                NDATA=8192
0014          555   FORMAT(10X,' NUMBER OF DATA POINTS EXTRACTED  =',I20,///)
0015                N=NDATA
0016          99    FORMAT(/,'1FREQ(HZ)       MAGNITUDE')
0017                NN(1)=N
0018                DF=1.0/(N*DT)
0019                N2=(FMAX/DF)*2
0020                IF(N2.GT.N) N2=N
0021                CALL FOURT(AMP,NN,1,-1,0,WORK,2500)
0022                DO 5 J=2,N,2
0023                X=AMP(J-1)
0024                Y=AMP(J)
0025           5    AMP(J-1)=SQRT(X*X+Y*Y)/N
0026                WRITE(6,99)
0027                IF(IDO.EQ.1) GO TO 50
0028                CALL PLOT(AMP,DF,DF,N1,N2,N3)
0029          50    F=DF
0030                IF (IDO.EQ.3) GO TO 51
0031                PRINT 100
0032          100   FORMAT(/,' FREQ(HZ)         MAGNITUDE')
0033                SUM=0.0
0034                J=1
0035                NC=0
0036                DO 10 I=N1,N2,N3
0037                NC=NC+1
0038                SUM=AMP(I) +SUM
0039                IF(NC.LT.N4) GO TO 10
0040                NC=0
0041                AMP(J)= SUM/N4
0042                SUM=0.0
0043                IF(F.GT.FM) GO TO 11
0044                J=J+1
0045          10    F=F+ DF
0046          11    DFP=N4*DF
0047                SF=(DF + DFP)/2.0
0048                CALL PLOT(AMP,SF,DFP,1,J,1)
0049          51    CONTINUE
0050                RETURN
0051                END
```

NDATA - Number of data points in AMP array.

N1 - Designates the first point to be plotted from the spectral data array by PLOT.

N3 - Directs PLOT to skip N3 points between each plotted point in spectral data array.

N4 - Number of points to be averaged when using the averaging feature of this routine.

IDO - If IDO = 1 the "average" feature is to be used. If IDO = 3 the "average" feature is not to be used.

N8 - Number of points (zeros) added by FILL.

SUBROUTINE PRINT (NNN)

This subroutine accepts the error cumulation array from EXT after a run is completed. It prints out the error table and other error data.

Labeled Common Variables

IE - Array containing the sum of 7 types of errors for each of the 300 records.

IRCEND - The number of the last record to be processed.

SUBROUTINE SIMP (SUM, FLL, FUL, N, NTYP, A, B, C)

This is a Simpsons rule integration routine called by CHI.

Argument Variables

SUM - Result of integration.

FLL - Lower limit.

FUL - Upper limit.

N - Number of points - 21.

NTYP - Determines if routine will compute for a normal test, log-normal test or both.

A - Mean value of amplitudes

B - Log standard deviation.

C - Mean value of log-amplitude.

```
FORTRAN IV G LEVEL 1, MOD 4            PRINT            DATE = 70122          02/35/57

0001                SUBROUTINE PRINT(NNN)
0002                COMMON/PR/IE(7,300)
0003                COMMON/RAT/IRCEND
0004                DIMENSION IESUM(7)
0005                PRINT 3
0006           3    FORMAT(1H1,50X,'DATA PROCESSING IRREGULARTIES',/,10X,'ERROR CODES
                   *FOLLOW',////,
                   *10X,'NO BASE POINTS FOUND IN BASE SEARCH **********  1'/,
                   *10X,'NUMBER OF BASE POINTS EXCEEDS 10  ***********  2'/,
                   *10X,'NUMBER OF SIGNAL POINTS EXCEEDS 10 **********  3'/,
                   *10X,'NUMBER OF BASE POINTS INSUFFICIENT **********  4'/,
                   *10X,'NUMBER OF SIGNAL POINTS INSUFFICIENT *********  5'/,
                   *10X,'NUMBER OF PASSES EXCEEDS LIMIT L1 ***********  6'/,
                   *10X,'NUMBER OF ERRORS IN RECORD EXCEEDS 5 *********  7')
0007                DO 555 I=1,7
0008            555 IESUM(I)=0
0009                PRINT 1
0010           1    FORMAT(10X,'ERROR',11X,'1',13X,'2',13X,'3',13X,'4',13X,'5',13X,
                   *'6',13X,'7')
0011                PRINT 100
0012           100  FORMAT(10X,'RECORD',///)
0013                DO 55 I=1,IRCEND
0014                DO 55 K=1,7
0015            55  IESUM(K)=IESUM(K)+IE(K,I)
0016                DO 2 I=1,IRCEND
0017                DO 12 K=1,7
0018                IF(IE(K,I)) 15,12,15
0019           12   CONTINUE
0020                GO TO 2
0021           15   WRITE(6,5) I,(IE(K,I),K=1,7)
0022           5    FORMAT(10X,I3,10X,I4,10X,I4,10X,I4,10X,I4,10X,I4,10X,I4,10X,I4)
0023              2 CONTINUE
0024                PRINT 20
0025           20   FORMAT(10X,//,10X,'ERROR CODE',10X,'NUMBER OF ERRORS')
0026                DO 50 K=1,7
0027           50   PRINT21,K,IESUM(K)
0028            21  FORMAT(10X,I6,14X,I9)
0029                RETURN
0030                END
```

```
FORTRAN IV G LEVEL 1, MOD 4          SIMP              DATE = 70122          02/35/57

0001                  SUBROUTINE SIMP(SUM,FLL,FUL,N,NTYP,A,B,C)
                C      INTEGRAND FUNCTION   REMOVE WHENCHANGING FUNCTION
0002                  PBF(X,A,S) = (1.0/(S*SQRT(6.28318)))*EXP(-0.5*(((X-A)/S)**2))
0003                  FNP=N-1
0004                  DELX=(FUL-FLL)/FNP
0005                  SUM=0.0
0006                  SUM1=0.0
0007                  SUM2=0.0
0008                  DO 1 I=1,N
0009                  FK=I-1
0010                  X=FK*DELX+FLL
                C     CALL FOR INTERGRAND SUBROUTINE HERE
                C     CALL FOR INTERGRAND SUBROUTINE HERE
0011                  IF(NTYP) 101,101,110
0012                101 VAL=PBF(X,A,B)
0013                  GO TO 102
0014                110 IF(X) 20,20,100
0015                 20 VAL=0.00
0016                  GO TO 102
0017                100 XX=0.5*ALOG(X/A)
0018                  VAL=PBF(XX,C,B)
0019                  VAL=0.5*VAL/X
0020                102 CONTINUE
                C
0021                  IF(I.EQ.1.OR.I.EQ.N) GO TO 2
0022                  J=MOD(I,2)
0023                  IF(J)3,4,3
0024                3 SUM1=SUM1+VAL
0025                  GO TO 1
0026                4 SUM2=SUM2+VAL
0027                  GO TO 1
0028                2 SUM=SUM+VAL
0029                1 CONTINUE
0030                  SUM=SUM+2.0*SUM1 + 4.0*SUM2
0031                  SUM=SUM*DELX/3.0
0032                  RETURN
0033                  END
```

SUBROUTINE FILL (I)

This routine is called by subroutine EXT in cases where data points are discarded due to irregularities. FILL places zeros into the omitted positions.

Argument Variables

I - Number of data points discarded.

Labeled Common Variables

AMP - Array containing extracted data.

NDATA - Number of data points in AMP.

N8 - Number of zeros added by FILL.

```
FORTRAN IV G LEVEL 1, MOD 4            FILL            DATE = 70122          02/35/57

0001                SUBROUTINE FILL(I)
0002                COMMON/AP/AMP(10000),NDATA
0003                COMMON/NPTS8/N8
0004                J=(I+3)/6
0005                N8=N8+J
0006                DO 1 K=1,J
0007                NDATA=NDATA + 1
0008                AMP(NDATA)=0.
0009                RETURN
0010                END
```

DATA PROCESSING IRREGULARTIES

ERROR CODES FOLLOW

NO BASE POINTS FOUND IN BASE SEARCH ********** 1
NUMBER OF BASE POINTS EXCEEDS 10 ********** 2
NUMBER OF SIGNAL POINTS EXCEEDS 10 ********** 3
NUMBER OF BASE POINTS INSUFFICIENT ********** 4
NUMBER OF SIGNAL POINTS INSUFFICIENT ********** 5
NUMBER OF PASSES EXCEEDS LIMIT L1 ********** 6
NUMBER OF ERRORS IN RECORD EXCEEDS 5 ********** 7

ERROR RECORD	1	2	3	4	5	6	7
2	0	0	0	1	0	0	0
232	0	1	0	0	0	0	0
240	0	0	0	1	0	0	0
241	0	0	0	1	0	0	0
252	0	0	0	1	0	0	0
253	0	0	0	1	0	0	0
264	0	0	0	1	0	0	0
265	0	0	0	1	0	0	0
276	0	0	0	1	0	0	0
277	0	0	0	1	0	0	0
288	0	0	0	1	0	0	0
289	0	0	0	1	0	0	0
300	0	0	0	1	0	0	0

ERROR CODE	NUMBER OF ERRORS
1	0
2	1
3	0
4	12
5	0
6	0
7	0

ERROR TABLE

TAPE NUMBER 3397 TRACK 4 FILE 2

AMPLITUDE	LOG AMPLITUDE	COUNT	CUMULATIVE PROBABILITY
0.235000E 03	-0.347886E 00	0.100000E 01	0.120077E-03
0.245000E 03	-0.327050E 00	0.200000E 01	0.360230E-03
0.255000E 03	-0.307047E 00	0.200000E 01	0.600384E-03
0.265000E 03	-0.287814E 00	0.0	0.600384E-03
0.275000E 03	-0.269293E 00	0.0	0.600384E-03
0.285000E 03	-0.251434E 00	0.200000E 01	0.840538E-03
0.295000E 03	-0.234191E 00	0.100000E 01	0.960615E-03
0.305000E 03	-0.217523E 00	0.300000E 01	0.132085E-02
0.315000E 03	-0.201393E 00	0.600000E 01	0.204131E-02
0.325000E 03	-0.185766E 00	0.800000E 01	0.300192E-02
0.335000E 03	-0.170614E 00	0.400000E 01	0.348223E-02
0.345000E 03	-0.155907E 00	0.170000E 02	0.552353E-02
0.355000E 03	-0.141620E 00	0.190000E 02	0.780499E-02
0.365000E 03	-0.127730E 00	0.210000E 02	0.103266E-01
0.375000E 03	-0.114216E 00	0.390000E 02	0.150096E-01
0.385000E 03	-0.101057E 00	0.660000E 02	0.229347E-01
0.395000E 03	-0.882359E-01	0.720000E 02	0.315802E-01
0.405000E 03	-0.757353E-01	0.127000E 03	0.468300E-01
0.415000E 03	-0.635396E-01	0.178000E 03	0.682036E-01
0.425000E 03	-0.516343E-01	0.286000E 03	0.102546E 00
0.435000E 03	-0.400058E-01	0.381000E 03	0.148295E 00
0.445000E 03	-0.286417E-01	0.513000E 03	0.209894E 00
0.455000E 03	-0.175301E-01	0.739000E 03	0.298631E 00
0.465000E 03	-0.666013E-02	0.929000E 03	0.410182E 00
0.475000E 03	0.397811E-02	0.110800E 04	0.543228E 00
0.485000E 03	0.143953E-01	0.122700E 04	0.690562E 00
0.495000E 03	0.245999E-01	0.117400E 04	0.831532E 00
0.505000E 03	0.346000E-01	0.855000E 03	0.934198E 00
0.515000E 03	0.444044E-01	0.434000E 03	0.986311E 00
0.525000E 03	0.540202E-01	0.105000E 03	0.998919E 00
0.535000E 03	0.634542E-01	0.900000E 01	0.100000E 01

NUMBER OF OVERFLOWS 0
NUMBER OF UNDERFLOWS 0

NUMBER OF CLASS INTERVALS = 31

AVERAGE	STANDARD DEVIATION	SKEWNESS	KURTOSIS
0.471235E 03	0.323860E 02	-0.646181E 00	0.163909E 01
-0.127374E-02	0.364458E-01	-0.903620E 00	0.353928E 01

NUMBER OF DATA POINTS 8328

CHI SQUARE= 0.542616E 06
NUMBER OF CLASS INTERVALS USED = 23

CHI SQUARE= 0.534447E 07
NUMBER OF CLASS INTERVALS USED = 23

TYPICAL COMPUTER OUTPUT FOR STATISTICAL ANALYSIS

TAPE NUMBER=3397 IDCAL=3 BASE=1 NUMBER OF CHANNELS=05 NUMBER OF SCANS=0200

TYPE BASE CALCULATION	*****	IBTYP	*****	2
TYPE STATISTICS CALLED FOR	*****	IFLAG	*****	4
NUMBER OF CLASS INTERVALS	*****	NCI	*****	100
NUMBER OF SCANS PER RECORD	*****	NOSCAN	*****	200
NUMBER OF CHANNELS ON TAPE	*****	NOCHAN	*****	5
TOLERANCE ON BASE LIMITS	*****	TOL1	*****	0.050
TOLERANCE ON SIGNAL LIMITS	*****	TOL2	*****	0.100
NUMBER OF SIGNAL AND BASE POINTS REQUIRED PER CYCLE	*****	TOL3	*****	3.000
NUMBER OF PASSES REQUIRED TO ABORT CYCLE	*****	L1	*****	10
NUMBER OF BASE POINTS REQUIRED FOR SIGNAL POINT SEARCH	*****	L2	*****	5
PUNCHED OUTPUT FLAG	*****	NPNCH	*****	1
PRINT ERROR TABLE FLAG	*****	IPRINT	*****	0
TYPE ANALYSIS ROUTINE WANTED	*****	ISSB	*****	2
INFORMATION FOR FFT ROUTINE	*****	FMAX	*****	40.000
	*****	DT	*****	0.006
	*****	FM	*****	30.000
	*****	N1	*****	3
	*****	N3	*****	2
	*****	N4	*****	5
	*****	IDO	*****	5

TYPICAL PARAMETER VALUES

APPENDIX B

This program generates a set of N random numbers having a log-normal distribution and a pre-selected mean and standard deviation. The program is in the form of a FORTRAN IV subroutine.

Theory: By definition a log-normal random deviate is one whose logarithms are normal random deviates. Thus if (X_i) is a set of log-normal random numbers then there must exist a set of normal random numbers (y_i) related to the X_i by

$$y_i = \ln X_i \qquad \text{B1}$$

Equation B1 may be generalized by the addition of appropriate scaling factors; i.e., we may let

$$y_i = a \ln X_i + b \qquad \text{B2}$$

Now by choosing the mean and variance of the (y_i) and the values of the scale factors a and b it is possible to generate a set of (X_i) having any desired mean and variance from a set of normal deviates (y_i). Solving B2 for X_i we have

$$X_i = \exp\left(\frac{y_i - b}{a}\right) \qquad \text{B3}$$

Since we wish to specify only two parameters, viz., the mean and standard deviation of the (X_i) it seems reasonable to assume that we will need only two parameters in equation B3. We therefore let a = 1 and take the mean of the (y_i) to be zero. B3 then becomes

$$X_i = \exp(-b) \exp(y_i) \qquad \text{B4}$$

taking the average of both sides of equation B4 we have

$$\overline{X} = \exp(-b)\ \overline{\exp(y_i)} \qquad \text{B5}$$

and also taking the second moment of (X_i) about zero

$$\overline{X^2} = \exp(-b)\ \overline{\exp(2y_i)} \qquad \text{B6}$$

the averages of the exponential functions in equation B5 and B6 can be evaluated easily

$$\overline{\exp(ny_i)} = (2\pi t^2)^{-1/2} \int_{-x}^{x} \exp(ny) \cdot \exp(y^2/2\sigma)dy \qquad \text{B7}$$

Combining equation B5, B6 and B7 we obtain expressions which may be solved for the scale factor b and the required standard deviation of the (y_i)

$$\sigma^2 = \ln(\mu/\overline{X}^2) \qquad \text{B8}$$

and

$$\exp(-b) = X\exp(-\sigma^2/2) \qquad \text{B9}$$

where μ is the second moment of the (X_i) about zero.

Program: The log-normal generator makes use of the normal random number generator included in the IBM Scientific Subroutine Package for the 360 computer. This routine (GAUSS) generates normal random numbers with any required mean and standard deviation. Coding for the program is shown in the accompanying listing. The argument list is as follows:

AVE - The required mean.

VAR - The required standard deviation.

Y - A vector of log-normal random numbers returned by the subroutine. Y is dimensioned by the calling program.

N - The number of random numbers to be generated.

IX - A "seed" required by GAUSS. IX must be a 5 digit odd integer.

Statements 3 to 6 compute the required standard deviation for the Gaussian-random numbers and the proper scaling factor. Statements 7 to 9 call GAUSS compute a log-normal random number from equation B5.

Fortran List for Log-Normal Generator

```
 1   SUBROUTINE LOGN(AVE, VAR, Y, N, IX)
 2   DIMENSION Y(1)
 3   VAR = VAR**2 + AVE **2
 4   SIG = ALOG(VAR/AVE**2)
 5   Z BAR = EXP(SIG/2.0)
 6   SIG = SQRT(SIG)
 7   DO 1   I = 1, N
 8   CALL GAUSS(IX, SIG, 0.0, X)
 9   1  V(I) = (AVE/Z BAR)*EXP(X)
10   RETURN
11   END
```

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